FOURTEEN-NODE MIXED STIFFNESS ELEMENT AND ITS COMPUTATIONAL COMPARISONS WITH TWENTY-NODE ISOPARAMETRIC ELEMENT

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Abstract

In this paper, a family of 3-dimensional elements different from isoparametric serendipity is developed according to the variational principle and the convergence criteria of the mixed stiffness finite element method[8,9,10]. For the new family, which is named mixed stiffness elements, the number of nodes on the quadratic (resp. cubic) element is not 20 (resp. 32) but 14 (resp. 26). Theoretical analysis and various computational comparisons have found the mixed stiffness element superior over the isoparametric serendipity element, especially a substantial improvement in computational efficiency can be achieved by replacing the 20 node-isoparametric element with the 14-node mixed stiffness element.

1. Introduction

For finite element analysis of a 3-dimensional continuum, an extensively used quadratic element is the 20-node isoparametric element. The element with the serendipity family as shape functions is sophisticated and has many advantages, but its practical use in large-scale problems always leads to very high requirement for computer memory capacity and to large amount of computational work.

The trouble is caused by excesive nodes over one element. Hence a question: Can another kind of distorted rectangular block elements with less than 20 nodes and the same accuracy be constructed? The history of the finite element method has indicated that it seems impossible to find any such element in the domain of standard finite elements. Recent advances of nonstandard finite element methods[1-3,5-9] make it possible to take another way and consequently give an affirmative answer to the question. By means of the mixed stiffness finite element method[8-10], a new family of curved elements is developed in the paper. A progressively increasing number of nodes and hence improved accuracy characterize each new member of the family. With the exception of the linear element, the number of nodes on each element of the new family is less than that of old serendipity family by six. For quadratic and cubic elements, they are 14 and 26 respectively.

Despite of the frequent use of linear and cubic elements in engineering design offices, we will focus our discussion, on account of the typicalness and practical importance of the quadratic element, on the 14-node mixed stiffness element. Through theoretical analysis and computational comparison with the 20-node isoparametric element, we will see that the performance of the new elements is quite satisfactory.

The paper is outlined as follows. In Sect. 2 the variational formulations of the mixed stiffness method are briefly introduced. Sect. 3 is devoted to description of the essentials of direct formulation. In Sect. 4 three convergence criteria of the mixed stiffness method are posed, and the approximation of the 14-node element examined. Sect. 5 is concerned with computational comparisons. Sect. 6 is the conclusion.

2. Mixed Stiffness Finite Element Method

Let us consider the basic boundary problem of elasticity

$$-B^T\left(\frac{\partial}{\partial x_i}\right)DB\left(\frac{\partial}{\partial x_i}\right)\tilde{u}=f, \text{ in } \Omega, \tag{2.1}$$

$$\tilde{u}\,|_{\Gamma_u}=u_0, \tag{2.2}$$

$$T(\tilde{u})\,|_{\Gamma_\sigma}=T_0, \tag{2.3}$$

where $B\left(\frac{\partial}{\partial x_i}\right)$ = matrix of differential operator defining the strain-displacement relations,

D = elastic matrix,

T = vector of surface tractions and $T(u)=B^T(\cos(n, x_i))\,DB\left(\frac{\partial}{\partial x_i}\right)u$, where n is the surface normal,

Γ_σ = boundary of the domain Ω over which tractions T_0 are prescribed,

Γ_u = boundary of Ω over which displacements u_0 are prescribed, and $\Gamma_u=\partial\Omega\backslash\Gamma_\sigma$,

f = distributed body force.

A variational formulation equivalent to the boundary-value problem can be expressed as[8, 9]

$$\begin{aligned}\Pi_R^*=&-\sum_n\left\{\iint_{\Omega_n^*}\left[\frac{1}{2}\sigma^TD^{-1}\sigma+f\cdot u\right]d\Omega+\oint_{\Gamma_{\sigma n}}T_0\cdot u\,ds\right.\\&\left.-\sum_j\left[\iint_{\Omega_n^*\cap\Omega_j}\sigma^T\left(B\left(\frac{\partial}{\partial x_i}\right)u\right)d\Omega-\oint_{\Gamma_{nj}}T\cdot u\,ds\right]\right\}\end{aligned} \tag{2.4}$$

= stationary,

where σ = stresses,

u = displacements assumed to satisfy the prescribed boundary conditions,

$\{\Omega_n^*\}$ = a subdivision of Ω associated with the stresses,

$\{\Omega_j\}$ = another subdivision of Ω associated with the displacements, such that for any pair $(\Omega_j,\ \Omega_n^*)$ of subdomains $\partial(\Omega_j\cap\Omega_n^*)\backslash\partial\Omega_n^*$ either is empty or runs through the interior of Ω_n^*,

$\Gamma_{nj}=\partial(\Omega_n^*\cap\Omega_j)\backslash\partial\Omega_n^*$.

Contrasting this with the Hellinger-Reissner Principle, which can be expressed as

$$\Pi_R=-\sum_n\left\{\iint_{\Omega_n^*}\left[\frac{1}{2}\sigma^TD^{-1}\sigma+f\cdot u-\sigma^T\left(B\left(\frac{\partial}{\partial x_i}\right)u\right)\right]d\Omega+\oint_{\Gamma_{\sigma n}}T_0\cdot u\,ds\right\}$$

= stationary,

we see the following differences between the two variational principles: