FINITE DIFFERENCE SOLUTIONS OF THE BOUNDARY PROBLEMS FOR THE SYSTEMS OF FERRO-MAGNETIC CHAIN*

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§ 1

In the classical study of one-dimnesional motion of ferro-magnetic chain, the so-called Landau-Lifschitz equation for the isotropic Heisenberg chain is of the form

$$s_t = s \times s_{xx} + s \times \boldsymbol{h}, \tag{1}$$

where $s=(s_1, s_2, s_3)$ is a 3-dimensional vector valued unknown function, $\boldsymbol{h}=(0, 0, h(t))$ and $h(t)$ is a constant or a function of t, "$\times$" denotes the cross-product operator of two 3-dimensional vectors.

Recently, a lot of works contributed to the study on the soliton solutions for Landau-Lifschitz equation, on the interactions of the soliton waves, on the properties of the infinite conservative laws and others[1-4]. The equation with the diffusion term

$$s_t = s \times s_{xx} + \nu s_{xx} \tag{2}$$

is called the spin equation. These systems also appear in the investigation of the problems of physics of the condensation state of medium. In [5] the periodic boundary problem and the initial problem for somewhat more general systems of ferro-magnetic chain

$$z_t = z \times z_{xx} + f(x, t, z) \tag{3}$$

are discussed, where $z=(u, v, w)$ and f are 3-dimensional vector valued functions. In [6], the boundary problems in rectangular domain $Q_T=\{0\leqslant x\leqslant l;\ 0\leqslant t\leqslant T\}$ for the system (3) are considered with one of the following boundary conditions (*): the first boundary condition

$$z(0, t) = z(l, t) = 0; \tag{4}$$

the second boundary condition

$$z_x(0, t) = z_x(l, t) = 0; \tag{5}$$

and the mixed boundary condition

$$z(0, t) = z_x(l, t) = 0 \tag{6}$$

or

$$z_x(0, t) = z(l, t) = 0 \tag{7}$$

and the initial condition

$$z(x, 0) = \varphi(x), \tag{8}$$

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where $\varphi(x)$ is a 3-dimensional vector valued initial function. The existence of the weak solutions of the appropriate problems for the system (3) of ferro-magnetic chain are established in [5, 6] by means of the method of vanishing of diffusion term in the corresponding spin system

$$z_t = \varepsilon z_{xx} + z \times z_{xx} + f(x, t, z). \tag{9}$$

It can be seen that the coefficient matrix of the terms of second order derivatives of the system (3) is zero-definite and is singular at $uvw=0$. So the system (3) can be regarded as a strongly degenerate parabolic system. The system (9) is a non-degenerate quasilinear parabolic system.

The purpose of this paper is to prove the solvability of the boundary problems (*), (8) for the system (3) of ferro-magnetic chain by the finite difference method. The symbol (*) denotes any given one of the boundary conditions (4), (5), (6) and (7).

Let us divide the rectangular domain Q_T into small grids by the parallel lines $x=x_j (j=0, 1, \cdots, J)$ and $t=t_n (n=0, 1, \cdots, N)$, where $x_j=jh$, $t_n=nk$ and $Jh=l$, $Nk=T$. We take the finite difference system

$$\frac{z_j^{n+1}-z_j^n}{k} = z_j^{n+1} \times \frac{\Delta_+\Delta_- z_j^{n+1}}{h^2} + f(x_j, t_{n+1}, z_j^{n+1}), \tag{3$_h$}$$

where $\Delta_+ u_j = u_{j+1} - u_j$ and $\Delta_- u_j = u_j - u_{j-1}$. The finite difference boundary conditions are as follows

$$z_0^n = z_J^n = 0; \tag{4$_h$}$$

$$z_1^n - z_0^n = z_J^n - z_{J-1}^n = 0; \tag{5$_h$}$$

$$z_0^n = z_J^n - z_{J-1}^n = 0; \tag{6$_h$}$$

$$z_1^n - z_0^n = z_J^n = 0, \tag{7$_h$}$$

where $n=1, 2, \cdots, N$. The finite difference initial condition is

$$z_j^0 = \bar{\varphi}_j, \quad (j=0, 1, \cdots, J), \tag{8$_h$}$$

where $\bar{\varphi}_j = \varphi(x_j)$ $(j=0, 1, \cdots, J)$ and $\bar{\varphi}_1 = \varphi(0)$ (or $\bar{\varphi}_{J-1} = \varphi(l)$) in the case of the boundary condition $z_1^n - z_0^n = 0$ (or $z_J^n - z_{J-1}^n = 0$).

Now we make the following assumptions for the system (3) of ferro-magnetic chain and the initial 3-dimensional vector valued function $\varphi(x)$.

(I) $f(x, t, z)$ is a 3-dimensional vector valued continuous function for $(x, t, z) \in Q_T \times \mathbb{R}^3$ and satisfies the condition of semiboundedness

$$(u-v)\cdot(f(x, t, u) - f(x, t, v)) \leqslant b|u-v|^2, \tag{10}$$

where $(x, t) \in Q_T$, $u, v \in \mathbb{R}^3$ and b is a constant.

(II) For $(x, t, z) \in Q_T \times \mathbb{R}^3$, there is

$$|f(x, t, z) - f(y, t, z)| \leqslant (A|z|^3 + B)|x-y| \tag{11}$$

for $x, y \in [0, l]$, $z \in \mathbb{R}^3$, $0 \leqslant t \leqslant T$, where $A \geqslant 0$ and $B \geqslant 0$ are constants.

(III) $\varphi(x)$ is a 3-dimensional vector valued continuously differentiable function in $[0, l]$ and satisfies the appropriate boundary condition (*).

The scalar product of two 3-dimensional vectors u and v is denoted by $u\cdot v$ and $|u|^2 = u\cdot u$. For the discrete vector valued functions $\{u_j\}$ and $\{v_j\}$, we take the notations, $(u\cdot v)_h = \sum_{j=0}^{J} (u_j\cdot v_j)h$ and $\|u\|_h^2 = (u\cdot u)_h$.