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STRONG PREDICTOR-CORRECTOR APPROXIMATION FOR STOCHASTIC DELAY DIFFERENTIAL EQUATIONS*

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Abstract

This paper presents a strong predictor-corrector method for the numerical solution of stochastic delay differential equations (SDDEs) of Itô-type. The method is proved to be mean-square convergent of order $\min\{1/2, \hat{p}\}$ under the Lipschitz condition and the linear growth condition, where $\hat{p}$ is the exponent of Hölder condition of the initial function. Stability criteria for this type of method are derived. It is shown that for certain choices of the flexible parameter p the derived method can have a better stability property than more commonly used numerical methods. That is, for some p, the asymptotic MS-stability bound of the method will be much larger than that of the Euler-Maruyama method. Numerical results are reported confirming convergence properties and comparing stability properties of methods with different parameters p. Finally, the vectorised simulation is discussed and it is shown that this implementation is much more efficient.

Mathematics subject classification: 65C30, 60H35.
Key words: Strong predictor-corrector approximation, Stochastic delay differential equations, Convergence, Mean-square stability, Numerical experiments, Vectorised simulation.

1. Introduction

In many scientific fields, such as biology, economics, medicine and finance, stochastic delay differential equations (SDDEs) are often used to model complex dynamics. Such equations generalize both deterministic delay differential equations (DDEs) and stochastic ordinary differential equations (SODEs). For the general theory on SDDEs, one can refer to Mao [22] and Mohammed [24].

Explicit solutions of SDDEs can rarely be obtained. Thus, it has become an important issue to develop numerical methods for SDDEs. In the last several decades, the research in

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the computational implementation and the numerical analysis for SODEs has made a lot of advances. An overview of these results can be found in some monographs and survey papers, see for example [1, 12, 13, 15, 25, 27].

The research into numerical methods for SDDEs is relatively new, compared with that for DDEs and SODEs. In recent years a number of numerical methods have been developed for SDDEs. For an introduction to the numerical analysis of SDDEs see Buckwar [7]. Baker & Buckwar [3] and Buckwar [8] derived several convergence results for one-step methods. Küchler & Platen [18] proposed the adapted low order Taylor methods for SDDEs. Moreover, for linear SDDEs, Baker & Buckwar [4], Cao, Liu & Fan [21] and Wang & Zhang [26] studied the stability properties of Euler-Maruyama method, semi-Euler method and Milstein method, respectively.

As in the deterministic case, using an explicit numerical scheme to solve a stiff system often results in instability and hence generates an inaccurate numerical solution. However, when an implicit method is used, the numerical stability and the computational accuracy can be greatly improved (cf. [14]). Hence, implicit numerical methods are preferred for the effective computation of numerical solutions to stiff systems. In the references [2, 17, 23], for solving stiff SODEs, the authors introduced implicitness into the approximation of the diffusion term and obtained several classes of the balanced implicit method. Here, an SODE is said to be stiff if it has widely varying lyapunov exponents. To implement an implicit method, generally speaking, an algebraic equation has to be solved at each time step, leading to a large computational cost. In order to resolve this difficulty, in papers [5, 6, 10], authors presented a few predictor-corrector schemes. Furthermore, Li et al. developed a family of strong predictor-corrector Euler-Maruyama methods for SODEs with Markovian switching, which were shown to converge with strong order 0.5 in [20]. But they did not take time delays into account. For SDDE with constant delay in Stratonovich form, Cao et al. [11] presented a predictor-corrector scheme using the Wong-Zakai approximation as an intermediate step, and proved the predictor-corrector scheme is of half-order convergence in the mean-square. This method was derived from the trapezoidal rule and does not have any free parameters. However, the performance of the predictor-corrector methods presented in this paper is tunable through the use of a free parameter p that controls the size of its stability region and hence the step size.

So far, to the best of our knowledge, no strong predictor-corrector scheme has been applied to SDDEs in Itô form. Hence in this paper we will focus on such a topic. We attempt to avoid implicit methods by using explicit methods with larger stability regions to deal with moderately stiff problems. The strong Euler predictor-corrector methods will be extended to solve SDDEs of Itô-type. The adapted method will be proved to be convergent of order $\min\{1/2, \hat{p}\}$ under the Lipschitz condition and the linear growth condition, where $\hat{p}$ is the exponent of Hölder condition of the initial function. We also investigate the asymptotic mean-square stability of the extended predictor-corrector method. Numerical stability criterion is derived which shows that this type of method preserves the asymptotic MS-stability of the underlying equation. Numerical examples will be given to illustrate these theoretical results. It is shown that for certain choices of the flexible parameter p the method presented here can have a larger stability bound than the Euler-Maruyama method. We also demonstrate that substantial speed-ups are possible by vectorising across the simulations the implementation of the numerical method.

2. The Strong Predictor-corrector Method

Let $W(t)$ be a one-dimensional standard Wiener process defined on the filtered probability space $(\Omega, \mathscr{A}, P)$, and $C([-\tau, 0]; \mathbb{R})$ denote the Banach space consisting of all continuous paths