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NEW TRIGONOMETRIC BASIS POSSESSING EXPONENTIAL SHAPE PARAMETERS*

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Abstract

Four new trigonometric Bernstein-like basis functions with two exponential shape parameters are constructed, based on which a class of trigonometric Bézier-like curves, analogous to the cubic Bézier curves, is proposed. The corner cutting algorithm for computing the trigonometric Bézier-like curves is given. Any arc of an ellipse or a parabola can be represented exactly by using the trigonometric Bézier-like curves. The corresponding trigonometric Bernstein-like operator is presented and the spectral analysis shows that the trigonometric Bézier-like curves are closer to the given control polygon than the cubic Bézier curves. Based on the new proposed trigonometric Bernstein-like basis, a new class of trigonometric B-spline-like basis functions with two local exponential shape parameters is constructed. The totally positive property of the trigonometric B-spline-like basis is proved. For different values of the shape parameters, the associated trigonometric B-spline-like curves can be $C^2 \cap FC^3$ continuous for a non-uniform knot vector, and C^3 or C^5 continuous for a uniform knot vector. A new class of trigonometric Bézier-like basis functions over triangular domain is also constructed. A de Casteljau-type algorithm for computing the associated trigonometric Bézier-like patch is developed. The conditions for G^1 continuous joining two trigonometric Bézier-like patches over triangular domain are deduced.

Mathematics subject classification: 65D07, 65D18.
Key words: Trigonometric Bernstein-like basis, Trigonometric B-spline-like basis, Corner cutting algorithm, Totally positive property, Shape parameter, Triangular domain.

1. Introduction

Trigonometric splines and polynomials have attracted widespread interest within CAGD (Computer Aided Geometric Design), particularly within curve design, see for example [17, 29, 30, 37, 39, 45–47, 50] and the references therein. In [18–22], some quadratic and cubic trigonometric polynomial splines with shape parameters were shown. In [25], a class of cubic trigonometric Bézier curves with two shape parameters was proposed, which is an extension of the cubic trigonometric Bézier curves with a shape parameter given in [20]. In [52], a class of C-Bézier curves was constructed in the space $\text{span}\{1, t, \sin t, \cos t\}$, where the length of the interval serves as shape parameter. The C-Bézier curves can exactly represent the ellipse and

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the sine curves. When the length of the interval tends to zero, the C-Bézier curves yield the classical cubic Bézier curves. Geometric interpretation of the change of the shape parameter for C-Bézier curves was given in [26]. In [6], it was pointed out that the critical length for the space $\mathrm{span}\{1, t, \sin t, \cos t\}$ is 2π, which means that Extended Complete Chebyshev (ECC) space for the space $\mathrm{span}\{1, t, \sin t, \cos t\}$ exists only on interval of length less than 2π. In [2], this restriction was overcome by substituting ECC-space with the Canonical Complete Chebyshev (CCC) space.

For modifications of the curve form, it is worth to study the practical methods for adjusting curves by using tension shape parameters. In [10], a kind of variable degree polynomial splines was constructed in the space $\mathrm{span}\{1, t, (1-t)^p, t^q\}$, where p, q are two arbitrary integers greater than or equal to 3 and used as tension shape parameters. In [31,32], this space was proved to be a Quasi Extended Chebyshev (QEC) space and studied by using the blossom approach. Later, in [15], the space $\mathrm{span}\{1, t, t^2, ..., t^{n-2}, (1-t)^p, t^q\}$ was investigated by using a direct approach based on elementary analytic properties of the space, where $p, q \geq n+1$ are any real numbers. Based on the fact that the space $\mathrm{span}\{1, t, t^2, ..., t^{n-2}, (1-t)^p, t^q\}$ is a QEC-space for any real numbers $p, q \geq n-1$ and $\max(p, q) > n-1$, the dimension elevation process for the space was further studied via blossom approach, see [33–36]. Recently, in [3], the totally positivity property of the variable degree polynomial spline basis was proved within the general framework of CCC-space. For the problems of shape preserving interpolation and approximation, the variable degree polynomial splines show great potential applications, see [11,13,14,16]. In [54], a kind of $\alpha\beta$-Bernstein-like basis with two exponential shape parameters was constructed in the space $\mathrm{span}\{1, 3t^2-2t^3, (1-t)^\alpha, t^\beta\}$, which forms an optimal normalized totally positive basis and includes the cubic Said-Ball basis and the cubic Bernstein basis as special cases. In [42], by using an iterative integral method, changeable degree spline basis functions were defined. In [44], the explicit representations of changeable degree spline basis functions were given. In [24], five trigonometric blending functions possessing two exponential shape parameters were constructed in the space $\mathrm{span}\{1, \sin t(1-\sin t)^{\alpha-1}, \cos t(1-\cos t)^{\beta-1}, (1-\sin t)^\alpha, (1-\cos t)^\beta\}$, based on which a class of trigonometric B-spline-like curves with three local shape parameters and a global shape parameter were proposed. In [27], these five trigonometric blending functions were further extended to a general case. It was pointed out in [12] that for constructing space curves, $C^2 \cap FC^3$ is a reasonable smoothness property since such continuity can ensure that the motion of a point on the generated curves have a continuous acceleration and that the generated curves possess continuous curvature vectors and torsion. And by using a modification of the C^4 quintic splines, a class of $C^2 \cap FC^3$ spline curves possessing tension shape properties was described in [12]. Based on the quartic Bernstein basis functions, a class of general quartic spline curves with three local shape parameters was proposed in [23]. The given spline curves can be $C^2 \cap GC^3$ continuous by fixing some values of the curves' shape parameters.

Since tensor product Bézier patch is the direct extension of Bézier curve, we can get rectangular patches with shape parameters through the above mentioned new curves. However, the Bernstein-Bézier surface over the triangular domain is not a tensor product patch exactly. Therefore, we cannot get triangular surfaces with an adjustable shape through the method of tensor product. During the last years, some researchers have put many efforts on the establishments of new bases over triangular domain with shape parameters, see for example [8,43,48,49,51,53]. In [43], Shen and Wang proposed a kind of Bernstein-like basis with a shape parameter, which was a triangular domain extension of the p-Bézier basis of order 3 given in [39]. In [48], Wei, Shen and Wang extended the C-Bézier basis on the univariate domain