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Feedback Control and Synchronization of Chaos for the Coupled Dynamos Dynamical System *

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Abstract: In this paper, by the simple linear controller, the coupled dynamos dynamical system can be controlled to a stable periodic orbit and a stable fixed point. Meanwhile, the stability of the period orbit and fixed point is proved by the values of Lyapunov exponent. Furthermore, by the nonlinear feedback controller, the efficient complete Synchronization of the coupled dynamos dynamical system is completed. Numerical simulation results show the effectiveness and feasibility of the simple linear controller and nonlinear controller.

Keywords: chaos, feedback control, efficient complete Synchronization, coupled dynamos dynamical system, Lyapunov exponent.

1. Introduction

In the last decade, controlling chaos has been the focus of the research of nonlinear chaotic systems. Recently, there has been increasing interest in the research of chaos control and synchronization. Various methods of chaos control and synchronization have been proposed in recent years. The current control algorithms can be classified into two main categories: feedback control and non-feedback control. Among these, linear feedback control is an important and effective method, based on the design of different controller has achieved a lot of satisfactory results (see [1]-[5]). At the same time, chaos synchronization also is an important topic, and has obtained a lot of availability results (see [6] - [7]).

The system that we will study in this paper comes from H.N. Agiza (see [1]). The system consists of two dynamos systems which are connected with each other so that the current generated by any one of them produces the magnetic field for the other. We denote the angular velocities of their rotors by ω_1, ω_2 and the current generated by x, y, respectively. Then with appropriate normalization of variables, the mathematical model equations for this system are:

$$\begin{cases} \dot{x} = \omega_1 y - \mu_1 x \\ \dot{y} = \omega_2 x - \mu_2 y \\ \dot{\omega}_1 = q_1 - \varepsilon_1 \omega_1 - xy \\ \dot{\omega}_2 = q_2 - \varepsilon_2 \omega_2 - xy \end{cases}$$

Where q_1 and q_2 are the torques applied to the rotors, and $\mu_1, \mu_2, \varepsilon_1, \varepsilon_2$ are positive constants representing dissipative effects. By setting $\varepsilon_1 = \varepsilon_2 = 0, q_1 = q_2 = 1$, the above system can be simplified as the following dynamical system:

$$\begin{cases} \dot{x} = -\mu x + y(z + \alpha) \\ \dot{y} = -\mu y + x(z - \alpha) \\ \dot{z} = 1 - xy \end{cases}$$

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This system is difference from the Lorenz system and L$\ddot{u}$ system and Chen system.

For this system in [1], H.N.Agize used the method of linear feedback control and bound feedback control, and chosen the following control input

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = -\begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

And

$$u(t) = \begin{cases} -u_0 & u(t) < u_0 \\ u(t) & -u_0 < u(t) < u_0 \\ u_0 & u_0 < u(t) \end{cases}$$

Such that the chaos of the coupled dynamos dynamical system can be controlled to the equilibrium point. Furthermore, the above method was be used and suitable control input is chosen to stabilize the unstable equilibrium points E_1, E_2. At the same time, in order to suppress the chaotic behaviors, H.N.Agize (see [1]) choice the following control input

$$u(t) = f_1 + f_2 \sin(\omega, t).$$

Added it to the second equation, so that the chaos of the dynamical system is controlled to the limit cycles. The numerical simulations showed the effect of the control.

The outline of this paper is as follows: Section 1 introduces the coupled dynamos dynamical system and gives its properties; Section 2 introduces the models and mathematical structure of the linear control for chaos; Section 3 by the simple linear controller, the coupled dynamos dynamical system can be controlled to a stable periodic orbit and a stable fixed point. Meanwhile, the stability of the period orbit and fixed point is proved by the values of Lyapunov exponent; Section 4 completes the efficient Synchronization of the coupled dynamos dynamical system by the nonlinear controller. The numerical simulation results will prove the correctness of the simple linear controller and nonlinear controller.

2. The coupled dynamos dynamical system

The coupled dynamos dynamical system introduced by H.N.Agiza (see [1]) is a nonlinear dynamical system. The system consists of two dynamos systems connected together so that the current generated by any one of them produces the magnetic field for the other. By simplification, finally, the coupled dynamos dynamical system can be written as follows

$$\begin{cases} \dot{x} = -\mu x + y(z+\alpha) \\ \dot{y} = -\mu y + x(z-\alpha) \\ \dot{z} = 1 - xy \end{cases} \tag{1.1}$$

Where α and μ are constant of the motion. When $\mu = 2, \alpha = 1$, it has a chaotic attractor as shown in Fig.1.The derivative of the flow (1.1) is given by

$$\nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = -2\mu < 0 \tag{1.2}$$

Where F= (F_1, F_2, F_3) = ($-\mu x + y(z+\alpha), -\mu y + x(z-\alpha), 1 - xy$).Then system (1.1) is a forced dissipative system similar to Lorenz system. But they are different from. Thus the solutions of the system (1.1) are bounded as $t \to \infty$ for positive values of α and μ. But in a sense defined by Vanecek and Celikovsky (see [8]), the Lorenz system satisfies the condition $a_{12}a_{21} > 0$, while the coupled dynamos dynamical system satisfies the condition $a_{12}a_{21} < 0$, Hence the coupled dynamos system and the Lorenz system are different types of system.

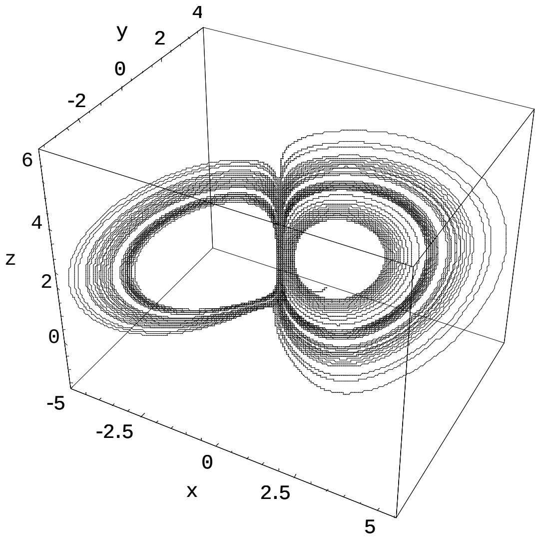

Fig.1 The chaotic attractor of the coupled dynamos dynamical system

3. Mathematical models of linear feedback control for chaos

Consider an N -dimensional system

$$\dot{x} = f(x, \lambda) \tag{2.1}$$

Where $x = (x_{1,} x_{2,} \cdots x_n)$, $\lambda = (\lambda_1, \lambda_2, \cdots \lambda_m)$ are the parameters of system (2.1).

In the following discussion, if the linear control input $u(x,k)$ is added to the right hand, thus the system

$$\dot{x} = f(x, \lambda) + u(x,k) \tag{2.2}$$

Can be considered as a controlled system, if $u(x,k) = 0$, the nonlinear system (2.1) have been considered as a chaotic system. By suitable choice $u(x,k)$, the chaotic system (2.1) can be controlled to a stable period orbit or a stable fixed point. In this paper, with this kind controlling models complete the control of the coupled dynamos dynamical system.

4. Linear feedback control of the coupled dynamos dynamical system

Consider the controlled coupled dynamos dynamical system

$$\begin{cases} \dot{x} = -\mu x + y(z+\alpha) \\ \dot{y} = -\mu y + x(z-\alpha) \\ \dot{z} = 1 - xy + k(y+z) \end{cases} \tag{3.1}$$

Where $u(x,k) = k(y+z)$ is a control input for the coupled dynamos dynamical system. If the parameter $k = 0$, then the above system is a chaotic system.

If $k = 0.265$, then the (3.1) can be rewritten as

$$\begin{cases} \dot{x} = -\mu x + y(z+\alpha) \\ \dot{y} = -\mu y + x(z-\alpha) \\ \dot{z} = 1 - xy + 0.265(y+z) \end{cases} \tag{3.2}$$

Where $\eta = \frac{1}{v}\frac{dv}{dt} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -3.735 < 0$ It can be proved that the system is a dissipation system, and the chaotic system (1.1) can be controlled to a stable period orbit. Moreover according to the A.Wolf, J.Swift,H.Swinney and J.Vastano(see[9]), we can obtain the Lyapunov exponents λ of the dynamical system(3.2) are 0.00051684, -0.40289 and -3.3272,respectly.the Lyapunov exponents graph as Fig.2.Thus the controlled chaotic coupled dynamos dynamical system (3.2) is a stable period orbit. By the numerical simulation we can obtain the graph of the stable period orbit as Fig.3.

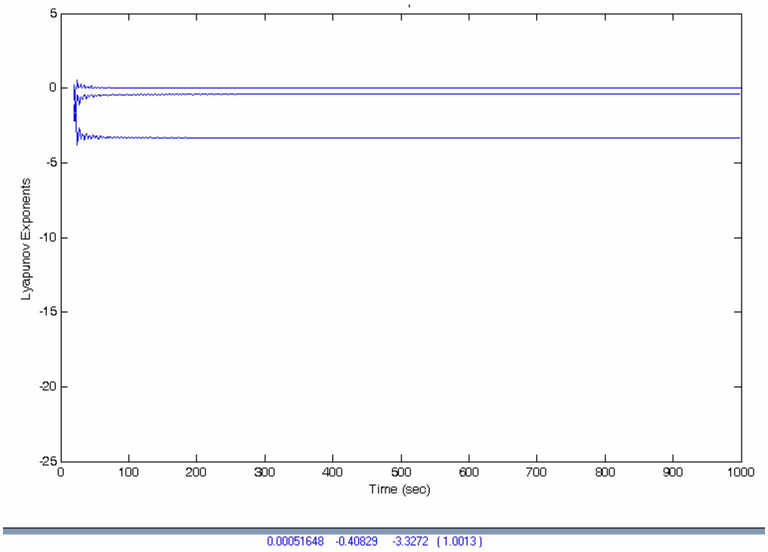

Fig.2 The evolution of the Lyapunov exponent of the controlled chaotic system

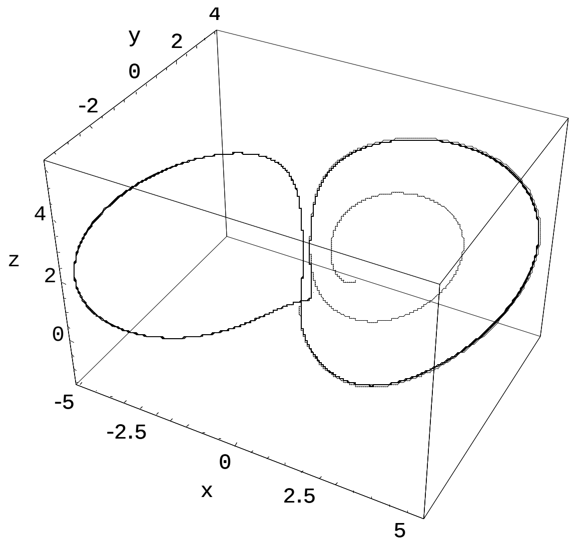

Fig.3 controlled stable periodic orbit

If we choice the controller as $u(x,k) = k(x+y+z)$ and then add it to the first equation of system (1.1), we can obtain the controlled system as following

$$\begin{cases} \dot{x} = -\mu x + y(z+\alpha) + k(x+y+z) \\ \dot{y} = -\mu y + x(z-\alpha) \\ \dot{z} = 1 - xy \end{cases} \tag{3.3}$$

When parameter $k = 0.066$, then the system (3.3) can be rewritten as

$$\begin{cases} \dot{x} = -\mu x + y(z+\alpha) + 0.066(x+y+z) \\ \dot{y} = -\mu y + x(z-\alpha) \\ \dot{z} = 1 - xy \end{cases} \tag{3.4}$$

Where $\eta = \frac{1}{v}\frac{dv}{dt} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -4.934 < 0$

Like the above, the system is a dissipation system, and the chaotic system (1.1) can be controlled to a stable fixed point. According to the A.Wolf, J.Swift,H.Swinney and J.Vastano(see[9]), we can obtain the Lyapunov exponents λ of the dynamical system(3.4) are -1.30785,-1.30789 and -12.5643,respectly.the Lyapunov exponents graph as Fig.4.Thus the controlled chaotic system (3.4) is a stable fixed point. By the numerical simulation, we can obtain the graph of the stable fixed point as Fig.5.

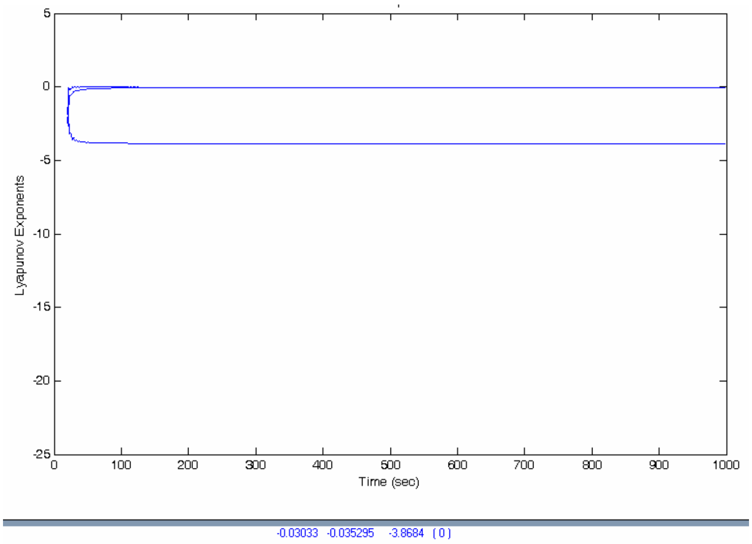

Fig.4 The evolution of the Lyapunov exponent of the controlled chaotic system

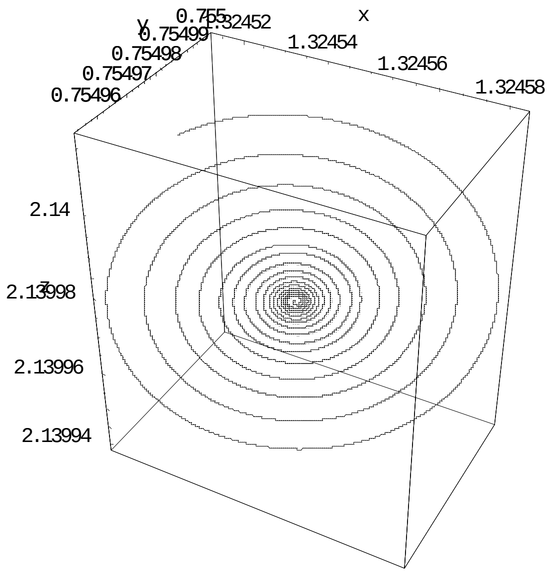

Fig.5 stable fixed point

5. Synchronization of the coupled dynamos dynamical system

Consider the system of differential equations

$$\dot{x} = f(x) \tag{4.1}$$

$$\dot{y} = g(x, y) \tag{4.2}$$

Where $x \in R^n, y \in R^n, f, g : R^n \to R^n$ are assumed to be analytic functions.

Let $x(t, x_0), y(t, y_0)$ be solutions to (4.1) and (4.2), respectively. The solutions $x(t, x_0), y(t, y_0)$ are said to be efficient complete synchronize if

$$\lim_{t\to\infty}\|x(t,x_0)-y(t,y_0)\|=c \tag{4.3}$$

Where c is a constant (see [7]). If $c=0$, then $x(t,x_0), y(t,y_0)$ are said complete synchronization (see [10]).

If we choice the drive system as

$$\begin{cases}\dot{x}_1=-\mu x_1+y_1(z_1+\alpha)\\ \dot{y}_1=-\mu y_1+x_1(z_1-\alpha)\\ \dot{z}_1=1-x_1y_1\end{cases} \tag{4.4}$$

Construct the respond system as

$$\begin{cases}\dot{x}_2=-\mu x_2+y_2(z_2+\alpha)+u_1\\ \dot{y}_2=-\mu y_2+x_2(z_2-\alpha)+u_2\\ \dot{z}_2=1-x_2y_2+u_3\end{cases} \tag{4.5}$$

Then the error between drive system and respond system is

$$\begin{cases}e_1=x_2-x_1\\ e_2=y_2-y_1\\ e_3=z_2-z_1\end{cases} \tag{4.6}$$

Choice the Lyapunov function

$$E=\frac{1}{2}(e_1^2+e_2^2+e_3^2) \tag{4.7}$$

when choice the control input as

$$\begin{cases}u_1=y_1z_1-y_2z_2+k_1e_1\\ u_2=x_1z_1-x_2z_2+k_2e_2\\ u_3=-x_1y_1+x_2y_2+k_3e_3\end{cases} \tag{4.8}$$

As long as $k_1>-\mu, k_2>-\mu, k_3<0$. Then the derivative

$$\dot{E}=-(\mu+k_1)e_1^2-(\mu+k_2)e_2^2+k_3e_3^2<0$$

So the error system is stable.

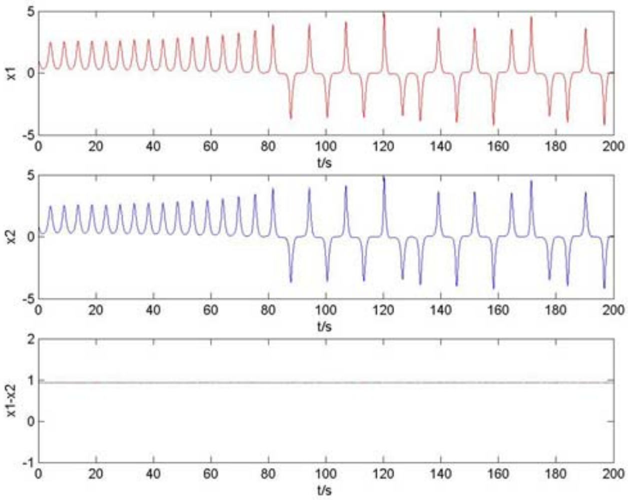

Fig.6 Time series of state variable $x_1(t)$, $x_2(t)$ and synchro error $x_1(t)-x_2(t)$

Choice $k_1 = -1, k_2 = 0, k_3 = -1$, $x_1(0) = 1$, $y_1(0) = 0, z_1(0) = 0$, $x_2(0) = 1$, $y_2(0) = z_2(0) = 1$,this particular choice will lead to the error states e_1, e_2, e_3 converge to constant as time t tends to infinity and hence the efficient synchronization is achieved. Numerical simulation results as Fig.6, Fig.7 and Fig.8.

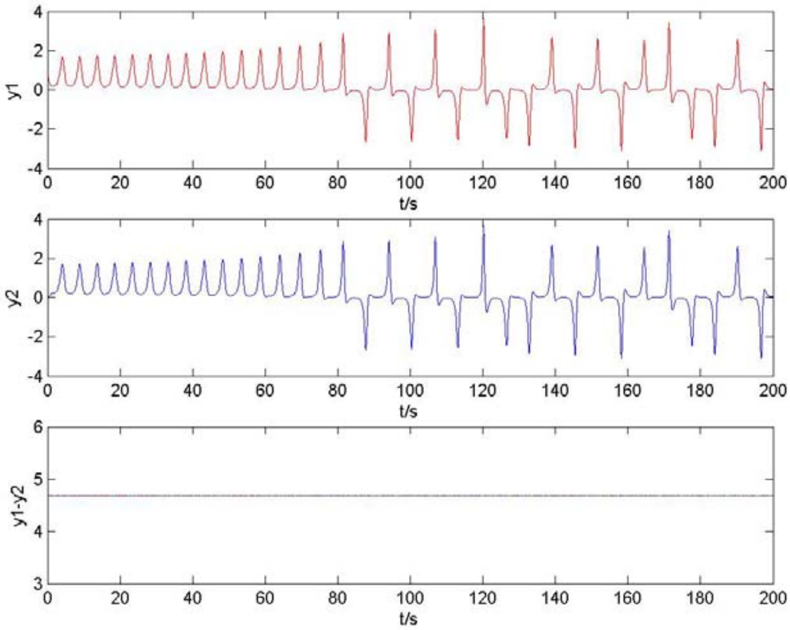

Fig.7 Time series of state variable $y_1(t)$, $y_2(t)$ and synchro error $y_1(t) - y_2(t)$

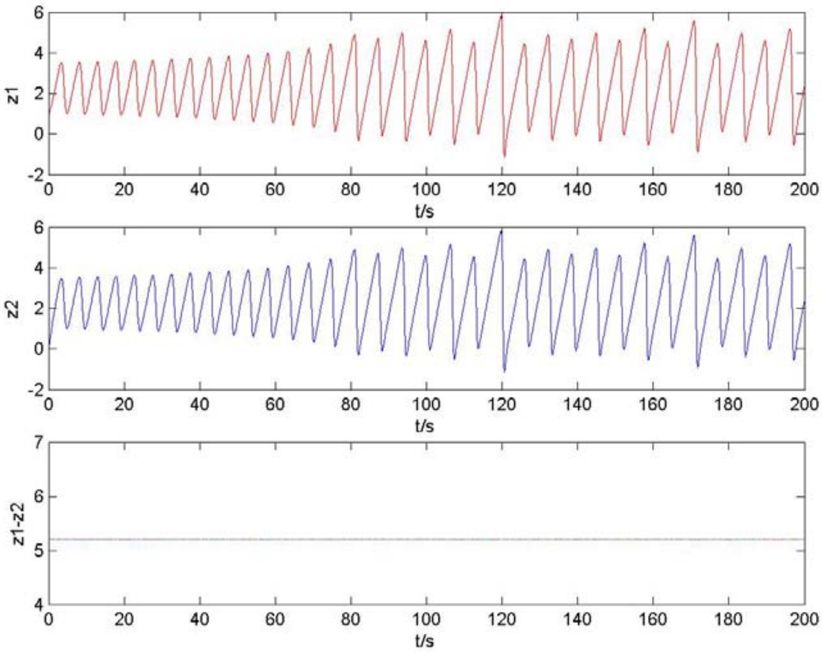

Fig.8 Time series of state variable $z_1(t), z_2(t)$ and synchro error $z_1(t) - z_2(t)$

6. Conclusion

In this paper, we use the simple linear controller, so the coupled dynamos dynamical system is controlled to a stable periodic orbit and a stable fixed point; by the nonlinear controller, the efficient synchronization is achieved. The Lyapunov exponent values and Lyapunov function prove the correctness of the methods that used in this paper. Actually, as long as chose a suitable control input u_1, u_2 and u_3, the complete synchronization between system (4.4) and (4.5) is also achieved.

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