

Fuzzy Model Identification: A Review and Comparison of Type-1 and Type-2 Fuzzy Systems

Meena Tushir

Department of Electrical & Electronics Engineering
Maharaja Surajmal Institute of Technology,
New Delhi, India.

Email: meenatushir@yahoo.com

(Received January 29, 2015, accepted May 27, 2015)

Abstract. Recently, a number of extensions to classical fuzzy logic systems (type-1 fuzzy logic systems) have been attracting interest. One of the most widely used extensions is the interval type-2 fuzzy logic systems. An interval type-2 TSK fuzzy logic system can be obtained by considering the membership functions of its existed type-1 counterpart as primary membership functions and assigning uncertainty to cluster centers, standard deviation of Gaussian membership functions and consequence parameters. This paper presents a review and comparison of type-1 fuzzy logic system and type-2 fuzzy systems in fuzzy modeling and identification. TSK fuzzy model is considered for both type-1 and type-2 fuzzy systems and model parameters are updated using gradient descent method. The experimental study is done on two widely known data, namely chemical plant data and the stock market data.

Keywords: Fuzzy Modeling, Identification, Type-1 Fuzzy Logic, Type-2 Fuzzy Logic.

1. Introduction

With the development of type-2 fuzzy logic systems (T2 FLSs) and their ability to handle uncertainty, interval type-2 FLCs (IT2 FLCs) has attracted a lot of interest in recent years. The concept of type-2 fuzzy sets was first introduced by Zadeh as an extension of the concept of well-known ordinary fuzzy sets, type-1 fuzzy sets. Mendel and Karnik have further developed the theory of type-2 fuzzy sets [1-5]. The type-2 fuzzy system has the capability to handle and minimize the effect of both linguistic and random uncertainties. A wide range of applications related to type-2 fuzzy system show that these systems provide much better solution specially in handling uncertainties. A type-2 fuzzy set [1-5] is characterized by a fuzzy membership function i.e. the membership grade for each element is also a fuzzy set in $[0,1]$, unlike a type-1 fuzzy set, where the membership grade is a crisp number in $[0,1]$. The membership functions of type-2 fuzzy sets are three dimensional and include a footprint of uncertainty (FOU), which is the new third dimension of type-2 fuzzy sets. The footprint of uncertainty provides an additional degree of freedom to make it possible to directly model and handle uncertainties. The FOU represents the blurring of a type-1 membership function, and is completely described by its two bounding functions, a lower membership function (LMF) and an upper membership function (UMF), both of which are type-1 fuzzy sets. Wu and Tan has shown in [6] that the extra degree of freedom provided by the footprint of uncertainty enables a type-2 FLS to produce outputs that cannot be achieved by type-1 FLSs with the same number of membership functions. Type-2 fuzzy sets are useful especially when it is difficult to determine the exact and precise membership functions.

Fuzzy systems are generally designed using either Mamdani or TSK type IF-THEN rules. In the former type, both the antecedent and consequent parts utilize fuzzy values. The TSK type fuzzy rules utilize fuzzy values in the antecedent part and crisp values or linear functions in the consequent part. The use of type-2 fuzzy systems for system identification is a recent topic and one can find few papers in the literature [7-10]. A recent work [7] presents a type-2 neuro fuzzy system for identification of time-varying systems and equalization of time-varying channels.

The paper is organized as follows. We start (Section 2) with a general overview of the type-1 fuzzy systems. A background of type-2 fuzzy systems is given in Section 3. The experimental results and comparative analysis are given in Section 4 followed by the main conclusions presented in Section 5.

2. Type-1 fuzzy logic systems

Fuzzy inference systems also known as fuzzy rule-based systems or fuzzy models are schematically shown in Fig. 1. They are composed of 5 conventional blocks: a rule-base containing a number of fuzzy IF-THEN rules, a database which defines the membership functions of the fuzzy sets used in the fuzzy rules, a decision-making unit which performs the inference operations on the rules, a fuzzification interface which transforms the crisp inputs into degrees of

match with linguistic values, a defuzzification interface which transforms the fuzzy results of the inference into a crisp output.

It is proved that Takagi-Sugeno fuzzy models are universal approximators of any smooth non-linear systems. In general, there are two approaches for constructing fuzzy models:

1. Identification (Fuzzy modeling using input-output data)
2. Derivation from given non-linear system equations.

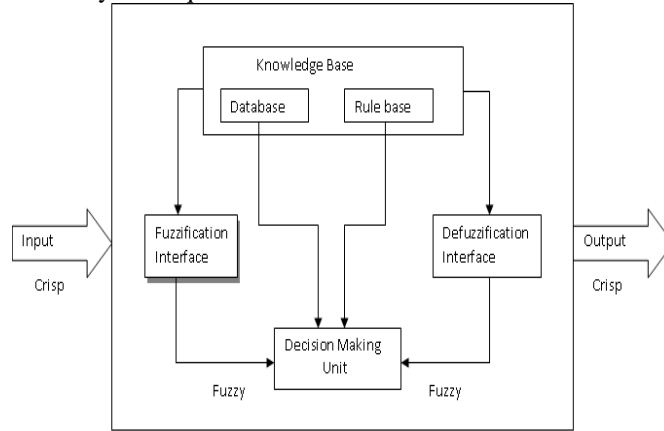


Fig 1: Type-1 Fuzzy Logic System

There has been an extensive literature of fuzzy modeling using input-output data following Takagi-Sugeno and Kang’s excellent work. The identification approach to fuzzy modeling is suitable for plants that are unable or too difficult to be represented analytically and/or by physical models. The procedure consists of two parts: structure identification and parameter identification.

A fuzzy model proposed by Takagi and Sugeno [11], is of the following form:

$$\text{Rule } i : \text{IF } x_1 \text{ is } A_{i1} \text{ and.....and } x_n \text{ is } A_{in} \text{ THEN } y_i = c_{i0} + c_{i1}x_1 + \dots + c_{in}x_n \tag{1}$$

Where $i=1,2,\dots,l$, l is the number of IF-THEN rules, c_{ik} ($k = 0,1\dots n$) are consequents parameters. y_i is an output from the IF-THEN rule, and A_{ij} is a fuzzy set.

Given an input (x_1, x_2, \dots, x_n) , the final output of the fuzzy model is inferred as follows:

$$y = \sum_{i=1}^l w_i y_i \tag{2}$$

where, y_i is calculated for the input by the consequent equation of the i^{th} implication, and the weight w_i implies the overall truth value of the premise of the implication for the input, and calculated as:

$$w_i = \prod_{k=1}^n A_{ik}(x_k) \tag{3}$$

where $A_{ik}(x_k) = \exp\left(-\frac{(x_k - a_{ik})^2}{b_{ik}^2}\right)$ is the Gaussian membership function. Here, a_{ik} and b_{ik} are parameters of the membership functions.

We used gradient descent technique to modify the parameters a_{ik} , b_{ik} and c_{ik} .

From Eqns (2) and (3), the overall output is given as:

$$y = \sum_{k=0}^n \sum_{i=1}^l w_i c_{ik} x_k \tag{4}$$

where, $x_0 = 1$.

The performance of the model is measured by the following index :

$$E = \frac{1}{2} (y^* - y)^2$$

where, y and y^* denote outputs of a fuzzy model and a real system, respectively. By partially differentiating E with respect to each parameter of a fuzzy model, we obtain: