Tracking the State of the Hindmarsh-Rose Neuron by Using the Coullet Chaotic System Based on a Single Input

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Abstract. Based on Lyapunov stability theory, a partial synchronization scheme is proposed to track the signal of Hindmarsh-Rose neuron using the Coullet system via only one single controller. Summation for the series of error variables are employed to detect the degree of synchronization. Three cases are considered to verify the proposed partial synchronization scheme. To demonstrate the effectiveness of the proposed method, some simulation results are given. It is found that the arithmetic product of the gain coefficients dominate the process and speed of synchronization of the two systems. The larger arithmetic product of the two gain coefficients is used, the less time is required.

Keywords: chaos; Hindmarsh-Rose system; Coullet system; partial synchronization.

1. Introduction

Chaos synchronization has attracted the attention of research community since Carroll and Pecora discovered it [1]. The problem has been studied due to its potential applications, such as in secure communication [2], in biological systems [3], in robotics [4]. Recently, synchronization is applied into complex networks [5, 6].

Chaos synchronization is to make two chaotic systems identical after transient initial states. For a sufficiently strong coupling, complete synchronization of chaotic systems can occur [7]. It is found that synchronization is useful and has many potential applications in many domains [2-6], especially, synchronization in physical or biological system is a fascinating subject attracting many renewed attention [5]. In recent years, synchronizations of coupled neuronal systems, such as Hindmarsh-Rose (HR) model[8,9], FitzHugh-Nagumo (FHN) model[10] and Chay model[11], have been studied.

In general, synchronization research has been focused on two areas: one is related with the employment of state observers, where the main applications lie on the synchronization of non-linear oscillators with the same structure and order, but different initial conditions and/or parameters[12–16]; the other is about the use of control laws to achieve the synchronization between non-linear oscillators with different structures and orders, where the variable states of the slave system are forced to follow the trajectories of the master system. The second approach can be seen as a tracking problem [17–21]. It transforms the tracking problem to a regulation problem with the origin (zero) as the corresponding set point. Some suitable controllers are designed to achieve the synchronization. Synchronous motion is often considered as the equality of corresponding variables of two systems. In other words, the trajectories of two systems will follow the same path after some transient. However, this situation is not the only commonly understood synchronization. Other different relationships between coupled systems can be considered synchronous. The concept of partial synchronization between two or more similar chaotic systems has been studied [22, 23]. Vieira and Lichtenberg [24] showed that partial (in their notation "weak") synchronization does not necessarily precede complete synchronization. Taborov et al. reported on partial synchronization in a system of three coupled logistic maps [25]. These results show that partial synchronization depends on the type of basic map constituting the coupled system. However, the mechanism of the occurrence of partial synchronization remains unclear.

In this paper, we will study the problem of tracking the **Hindmarsh-Rose neuron** signal by controlling the **Coullet chaotic system** with improved adaptive control scheme via single controller. In this scheme, changeable gain coefficients are introduced into Lyapunov function, which is composed of error variable between the outputs and the external standard signal. The controller is approached analytically. The main

difference between the scheme and the previous ones is that the gain coefficient in the controller and Lyapunov function is changeable with time. Three cases are considered and some numerical simulations are presented to demonstrate the effectiveness of the proposed scheme.

2. System description

2.1. Hindmarsh-Rose neuron system

The **Hindmarsh-Rose (HR)** neuronal model was first proposed by Hindmarsh and Rose as a mathematical representation of the firing behavior of neurons. It was originally introduced to give a bursting type with long inters pike intervals of real neurons [26]. The form of HR system is given by

$$\dot{x}_1 = ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext},$$
$$\dot{x}_2 = c - dx_1^2 - x_2,$$
$$\dot{x}_3 = r(S(x_1 + k) - x_3), \tag{1}$$

where a, b, c, d, r, S, k, I_{ext} are real constants.

2.2. Coullet chaotic system

The Coullet chaotic system, proposed by Coullet and Arneodo [27], is one of paradigms of chaos for it captures many features of chaotic systems. It includes a simple cube part and three simple ordinary differential equations that depend on three positive real parameters. The dynamic equations of the system can be written as following:

$$\dot{y}_1 = y_2,$$
$$\dot{y}_2 = y_3,$$
$$\dot{y}_3 = a_1 y_1 - a_2 y_2 - y_3 - y_1^3, \tag{2}$$

where a_1, a_2 are real constants.

3. Control schemes

In this section, we will propose a systematic design procedure to simulate the bursting activity of HR neuron system by using the Coullet system and the control scheme can be approached via an improved adaptive track. The mechanism can be understood as partial synchronization between dynamical models. This method needs only one single controller. A single control input u_1 is added to the second equation of system (2) and the first state y_1 of the controlled Coullet chaotic model is used to simulate the dynamical properties of the HR neuron system state. Thus, the controlled Coullet chaotic system is given as following

$$\dot{y}_1 = y_2,$$
$$\dot{y}_2 = y_3 + u_1,$$
$$\dot{y}_3 = a_1 y_1 - a_2 y_2 - y_3 - y_1^3, \tag{3}$$

The error is denoted as

$$e = x - y_1, \tag{4}$$

where x is one of the observed states of system (1). The error function $e = x - y_1$ will be stabilized to certain value when the output variables x and y_1 is close to each other, which indicates a kind of partial synchronization. To realize the partial synchronization between the two systems is now transformed to how to choose a control law u_1 and make e generally converge to zero with time increasing.

To realize partial synchronization, Lyapunov function is chosen as following:

$$V = \alpha e^2 + (\dot{e} + \beta e)^2, \tag{5}$$

where α and β are positive gain coefficients, the over dot denotes the differential variable e of time. The differential coefficient of the above Lyapunov function as shown in Eq.(5) of time is approached by