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Partial eigenvalue assignment in descriptor systems via derivative and propositional state feedback

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Abstract. A method for solving the descriptor continuous-time linear system is focused. For easily, it is converted to two standard continuous-time linear systems by the definition of a derivative and propositional state feedback. Then partial eigenvalue assignment is used for obtaining the derivative and propositional state feedbacks and solving the standard systems. In partial eigenvalue assignment, just a part of the open loop spectrums of two standard linear systems are reassigned, while leaving the rest of the spectrum invariant and for reassigning, similarity transformation is used. Using partial eigenvalue assignment is easier than using eigenvalue assignment. Because by partial eigenvalue assignment, size of matrices and state and input vectors are decreased and stability is kept, too. It is worthy to mention that eigenvalues of closed-loop matrix of original system, i.e., descriptor and second converted system are inverse of each other. Also concluding remarks and an algorithm are proposed to the descriptions be obvious. At the end, convergence of state and input vectors in the descriptor system to balance point (zero) are showed by figures in a numerical example.

Keywords: descriptor system, derivative and propositional state feedback, partial eigenvalue assignment, converge to balance point.

1. Introduction

Descriptor systems that are also called singular systems are more general and precise than a normal model to depict a dynamical physical. Applications of descriptor systems can be found in various fields such as artificial neuron networks, circuits systems, chemical processes, economics, biologic, power, modeling of mechanical multi body systems and etc. [6, 10, 11, 20, 25, 28]

Some of the first fundamental works on eigenstructure assignment in descriptor linear systems were established in the 1980s by a number of researchers, such as Cobb (1981) [5], Armentano (1984) [1], Fletcher (1986) [9], Ozcaldiran and Lewis (1987) [21].

In recent years there are many subjects are related to these problems like switched descriptor systems and eigenvalue assignment in state feedback control for uncertain systems [22, 27]. Also Karbassi et al. worked on non-linear state feedback controllers like in [19].

In the available literature on descriptor systems, there are two kinds of stabilization problems for singular systems. One consists in designing a state or output feedback controller in such a way that the closed-loop system is regular, impulse-free, and stable or equivalently admissible. The other is to design a state or output feedback controller in order to make the closed-loop system regular and stable. Concerning the stability analysis and the stabilization problem, a number of approaches assuming or not assuming the regularity of the descriptor system have been proposed in the literature let us quote for instance [2, 6, 26] among those assuming the regularity and [6, 26] without assuming the regularity. Also positivity and stability of linear descriptor systems have been investigated in [13, 15] for systems with regular pencils.

Many practical applications such as the design of large and sparse structures, electrical networks, power systems, computer networks, etc., give rise to very large and sparse problems and the conventional numerical methods for EVA problem do not work well. Furthermore, in the most of these applications only a small number of eigenvalues, which are responsible for instability and other undesirable eigenvalues, need to be reassigned. Clearly, a complete EVA, in case when only a few eigenvalues are bad, does not make sense. This consideration gives rise to the partial eigenvalue assignment (PEVA) problem for the linear control system such that undesirable eigenvalues are reassigned and other eigenvalues unaltered. An explicit solution to the partial eigenvalue problem by using one of orthogonality relations between eigenvectors for matrix polynomial is considered in [23]. The conditions for existence and uniqueness of the solution for the single-input problem were given in [24] and for multi-input were presented in [8].

In this paper, the stability of descriptor continuous-time linear systems will be investigated. Our method is mixed of PEVA, EVA by similarity transformation and a useful method for converting the descriptor linear system to the standard linear systems. First, the descriptor continuous-time linear system (1) is converted to the standard continuous-time linear systems (8) and (10) by the definition of the derivative and propositional state feedback (5) that are calculated by the PEVA method (section 3). In other hand, we need to reassign undesired eigenvalues of open-loop spectrums in standard systems with smaller sizes of matrices such that other eigenvalues unchanged. Also a theorem for existence and uniqueness solution for PEVA in multi-input is represented. Then feedbacks in descriptor system are obtained by an easy relationship between these feedbacks and gained feedbacks by PEVA from (21). It is important to say for reassigning undesired eigenvalues, similarity transformation (section 4) is used that is a simple method with high accuracy.

As mentioned it is clear that our method has some advantages that solving the descriptor continuous-time linear systems will be more and more easier. The first advantage is, converting descriptor continuous-time linear system to standard continuous-time linear systems, because working on standard systems is easier than descriptor systems. Also we do not need the assumption of being full rank open-loop matrix in standard systems because of using derivative and propositional state feedback [3]. EVA have been an applicable method for finding the solution in standard systems and their stability, but by PEVA just by reassigning a part of open-loop matrix spectrum in standard systems while keeping other eigenvalues unvariant, their stability are kept. In PEVA we decrease the size of matrices and state and input vectors, it is obvious that calculating is more easily than EVA and obtaining state feedback is so comfortable by state feedback governed in PEVA and they are other advantages of our method. Therefore the state and input vectors in the original system, i.e., descriptor continuous-time linear system, converge to balance point and we show this by figures in our example. It is worthy to mention that we do not need some assumptions like no having eigenvalues near zero and some criteria on some vectors and being distinct eigenvalues by orthogonality relations for PEVA in [23] or dealing with full row rank matrices in every performed algorithm and finding index of Shuffle and Drazin for descriptor systems in [4, 12, 14, 16] and they are other excellence of method in this paper. Also this method can be used for discrete-time descriptor linear systems by defining a suitable state feedback.

This paper is organized as follows. Next section, presents converting the descriptor continuous-time linear system to the standard continuous-time linear systems that the closed-loop matrix of the second standard system, i.e., (10) has inverse of eigenvalues of closed-loop matrix in original system, i.e., descriptor system (1). The PEVA problem for obtaining the derivative and propositional state feedbacks is displayed in section 3. Section 4 proposes the similarity transformation for reassigning eigenvalues in PEVA. An algorithm and numerical results are presented in section 5 by an algorithm with all proposed details in its previous sections and numerical examples with the results of all steps of algorithm in it. Also convergence of state and input vectors to balance point, i.e. zero, by their figures are showed. At final section, conclusion is given.

The following notation will be used: $\Re$ - the set of real numbers, $\mathbb{C}$ - the set of complex numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices and $\Re^{m} = \Re^{m\times 1}$, A^T - the transposed matrix of A, $\Omega(A)$ - spectrum of eigenvalues of the matrix A, I_n - the unit matrix of size n.

2. Statement of the problem

Consider the descriptor linear time-invariant controllable system of the form

$$E\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $E \in R^{n\times n}$ with $rank\,(E) \leq n$, $x(t) \in R^n$ is state vector and $u(t) \in R^m$ is input vector. It is assumed that $1 \leq m \leq n$, $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are open-loop and input matrices respectively. Also $x(0) = x_0$ is a nonzero definite vector.

The aim is the eigenvalue assignment to design a derivative and propositional state feedback controller matrix which produce a closed-loop system of (1) with a satisfactory response by shifting controllable poles $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$ from undesirable to desirable locations where $\lambda_i \in \mathbb{C}$ and $\lambda_i \neq 0$ and are self-conjugate complex numbers for $i = 1,2,...,n$ and by using the method in section 3 means PEVA we reassign p eigenvalues which $p \leq n$ while other eigenvalue of open-loop matrix unchanged.

As a brief displaying, we discuss the advantage of using the derivative and propositional state feedback controller instead of the derivative state feedback.