ISSN 1746-7659, England, UK
**Journal of Information and Computing Science**
Vol. 16, No. 2, 2021, *pp*.083-090

# Dynamics Analysis of a Kind of Chaotic System under Periodic Excitation

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(Received April 21, 2021, accepted September 12, 2021)

**Abstract:** The dynamical behaviors of the novel four-dimensional memristor self-oscillated system with periodic excitation are discussed. With the change of amplitude and angular frequency of external periodic excitation, the proposed system can generate various dynamics including chaotic, periodic and bursting oscillations. Using the numerical simulation method, the phase trajectory and time series are employed to verify the behavior of the considered system.

**Keywords:** Chaos system, Periodic excitation, Dynamical behavior

## 1. Introduction

Distinct from classical attractors, such as Lorenz attractor [1], Chen attractor [2], Lü attractor [3], and others [4-6], hidden attractor is a new type of attractor [7-11]. It was found that some chaotic systems without equilibrium or stable equilibrium can appear chaotic attractor. The hidden attractor has attracted much attention [12-15]. Additionally, the hidden attractor is often unwanted in many cases of practical application. To avoid or utilize the hidden attractor, it is meaningful and important to investigate the characteristic of it [16-21].

In recent years, a variety of chaotic systems with memristor have attracted much attention [22-26]. Bao et al. presented a new type of four-dimensional self-oscillated system with no equilibrium[22]. The coexistence of various hidden attractors including periodic solution, quasi-periodic periodic solution, chaotic attractors was studied. The Chaotic system could also display abundant dynamical behaviors under external stimulation. Wang et al. described a time-delay memristive Hindmarsh-Rose neuron model, and the transition of electrical activities of the neuron was investigated with the change of noise intensity[27]. Li et al. presented a Duffing system with periodic excitation and theoretically analyzed the dynamic phenomena of the system under different parameters [28]. For chaotic system with periodic excitation, abundant dynamical behaviors were investigated because of extra periodic excitation [29-31]. It is useful to explore the dynamics under periodic excitation. In this paper, variety of dynamical behaviors of the novel four-dimensional memristor self-oscillated system is discussed under external periodic excitation. With the change of amplitude and angular frequency of periodic excitation, the dynamics of proposed system is discussed, including chaotic, periodic, and bursting oscillations as well as coexistence of different dynamics. Other parts of this paper are arranged as follows. Section 2 presents a chaotic system with periodic ex-citation. The various dynamics of system are revealed in Sections 3. Conclusion are given in Section 4.

## 2. System description

A four-dimensional chaotic system is presented in Ref.1, including two linearly coupled term, one constant term and four nonlinear terms. The equations are described as

$$\begin{cases} \dot{x} = y \\ \dot{y} = (z + x^2 - \beta x^4)y - {\omega_0}^2 W(w)x \\ \dot{z} = \mu - x^2 \\ \dot{w} = x - w \end{cases}, \quad (1)$$

where $\beta$, $\mu$ and $\omega_0$ are parameters controlling the dynamics of system (1), $W(w) = a + b|w|$ is non-ideal voltage-controlled memristor with $a$,$b$ being intrinsic parameter [22]. To investigate the dynamics of system (1) under periodic excitation, system (1) can be revised as

$$\begin{cases} \dot{x} = y + Asin(\omega t) \\ \dot{y} = (z + x^2 - \beta x^4)y - \omega_0{}^2 W(w)x \\ \dot{z} = \mu - x^2 \\ \dot{w} = x - w \end{cases}, \tag{2}$$

where $A$ and ω are the amplitude and angular frequency of periodic excitation, respectively. It is obvious that system (2) is a nonautonomous system.

## 3. Dynamics analysis of the proposed system

With appropriate parameters, system (1) can exhibit chaotic attractors or periodic solution. In this section, the dynamics of system (2), namely, system (1) under periodic excitation, is to be explored with the change of amplitude and angular frequency of periodic excitation. For this end, we suppose that $\omega_0$ and $A$ change, respectively, while other parameters are fixed as $\beta$=0.5, $\mu$=0.9, $a$=1, $b$=0.1, $\omega = \omega_0$.

As the parameter $\omega_0$ is chosen as 1.68, chaotic attractor of system (1) is generated under initial values (-2,0,0,0) [22]. The phase portraits in $x - y$ plane is displayed in Fig.1.

Keeping other parameters and initial condition constant, the dynamic of system (2) is calculated via choosing differen value of stimulus amplitude and depicted in Fig.2. From Fig.2, it can be obtained that period-3 behavior is presented when $A = 0.4$; when $A = 0.6$, the period-1 behavior will appear. Thus, the system presents different dynamics as the amplitude changing. To better illustrate this, we also discuss other scenarios.

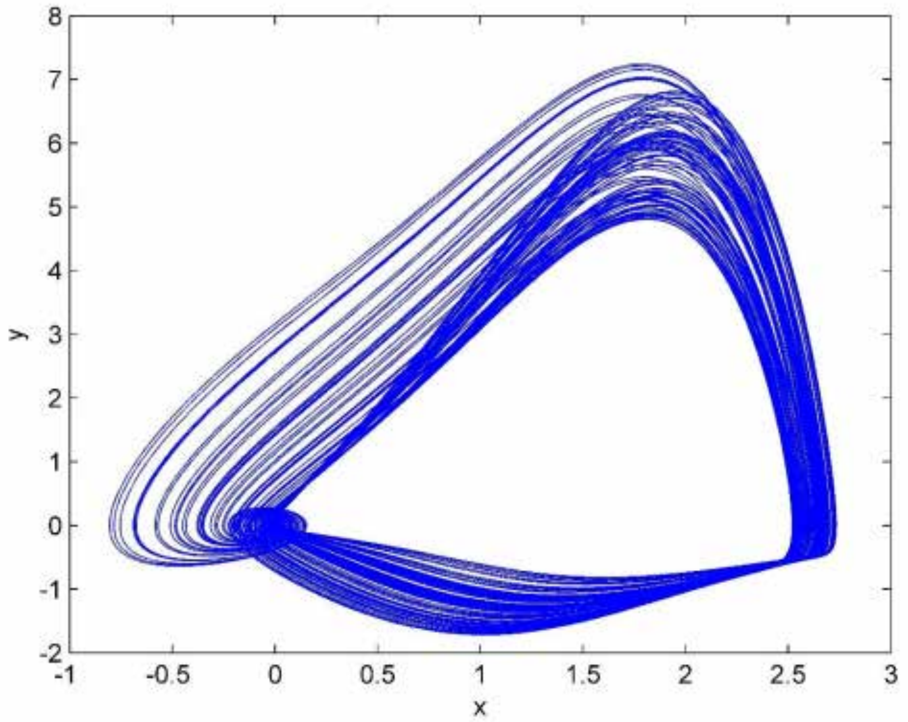


Fig.1. Phase portrait of system (1) in $\boldsymbol{x - y}$ plane with initial condition (-2,0,0,0) and $\boldsymbol{\omega_0}$=1.68.

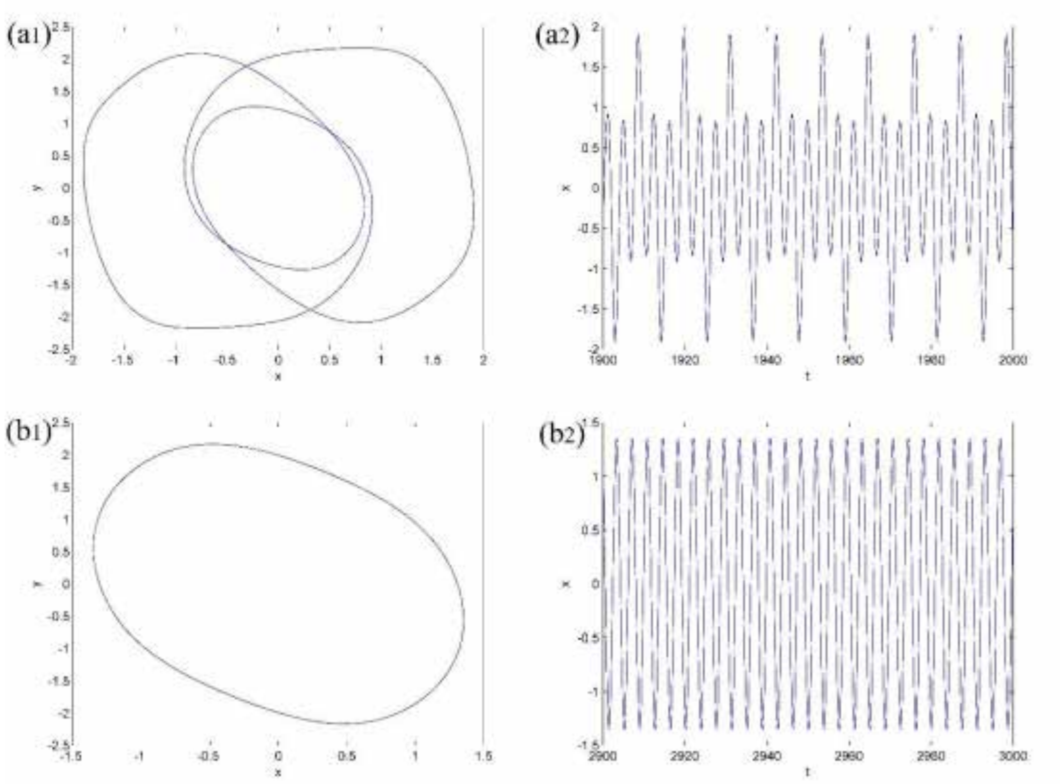


Fig.2. Phase portraits in $\boldsymbol{x}-\boldsymbol{y}$ plane and time series for variable $\boldsymbol{x}$ of system (2) with initial condition (-2,0,0,0) and $\boldsymbol{\omega_0} = \mathbf{1.68}$. (a1) Phase portrait in $\boldsymbol{x}-\boldsymbol{y}$ plane for $\boldsymbol{A} = \mathbf{0.4}$, (a2) Time series of variable $x$ for $\boldsymbol{A} = \mathbf{0.4}$, (b1) Phase portrait in $\boldsymbol{x}-\boldsymbol{y}$ plane for $\boldsymbol{A} = \mathbf{0.6}$, (b2) Time series of variable $x$ for $\boldsymbol{A} = \mathbf{0.6}$.

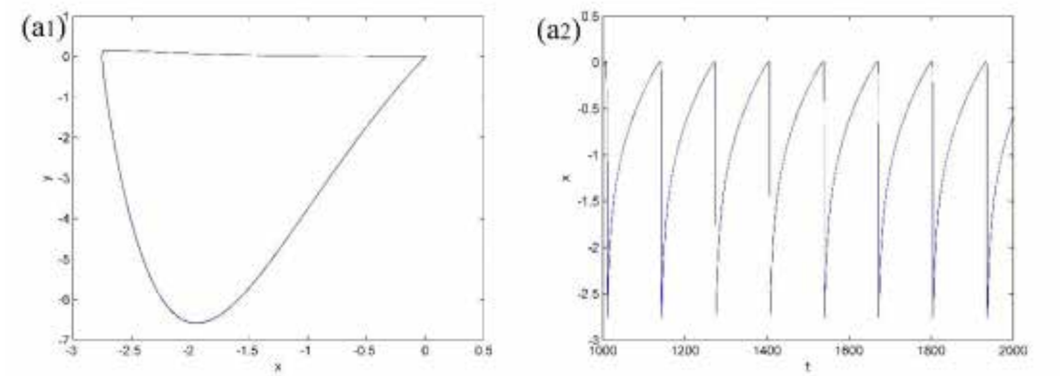


Fig.3. Phase portrait and time series for variable $\boldsymbol{x}$ of system (1) with initial condition (-2,0,0,0) and $\boldsymbol{\omega_0} = \mathbf{1}$, (a1) Phase portrait in $\boldsymbol{x}-\boldsymbol{y}$ plane; (a2) Time series of variable $x$ .

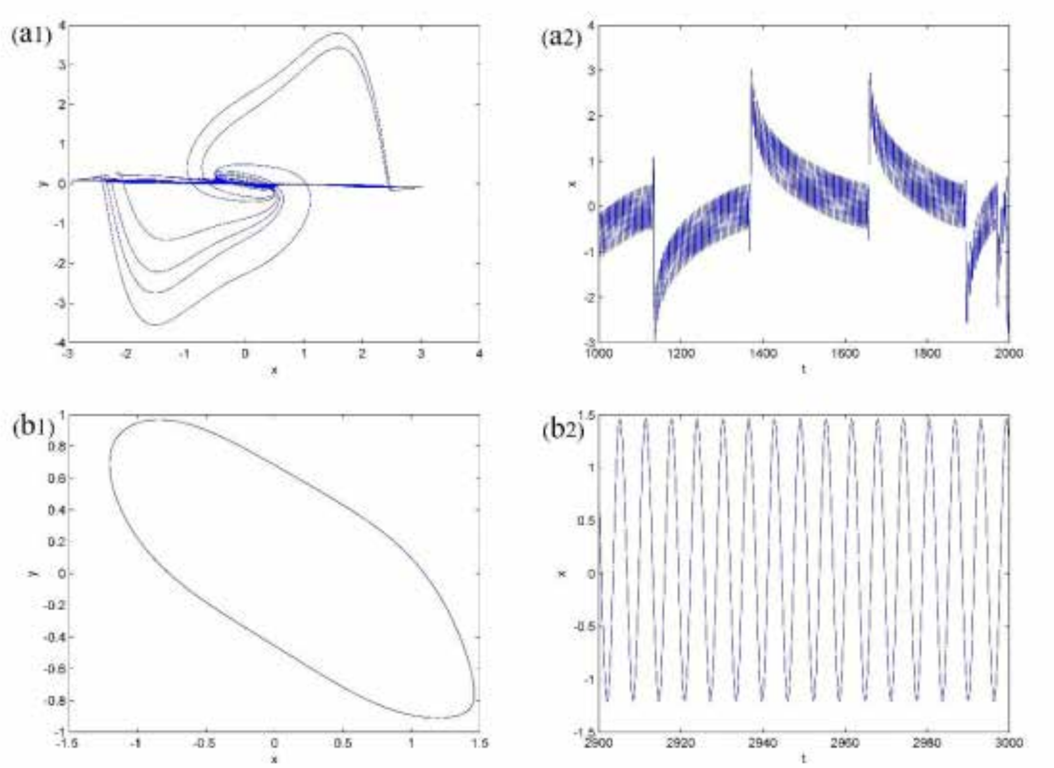


Fig.4. Phase portraits and time series for variable $\boldsymbol{x}$ of system (2) with initial condition (-2,0,0,0) and $\boldsymbol{\omega_0 = 1}$. (a1) Phase portrait in $\boldsymbol{x - y}$ plane for $\boldsymbol{A = 0.5}$;(a2) Time series of variable $x$ for $\boldsymbol{A = 0.5}$; (b1) Phase portrait in $\boldsymbol{x - y}$ plane for $\boldsymbol{A = 1}$; (b2) Time series of variable $x$ for $\boldsymbol{A = 1}$.

The system (1) appears period-1 behavior under initial conditions (-2,0,0,0) while $\omega_0 = 1$[22]. The phase portrait in $x - y$ plane and time series for variable $x$ are plotted in Fig.3. The bursting oscillations emerge when the amplitude of external stimulation is selected as 0.5, as shown in Fig.4. Fig.4(b1, b2) suggests that the system (2) presents period-1 but with different phase trajectories from Fig.3.

System (1) shows period-2 behavior under initial conditions (-2,0,0,0) as $\omega_0 = 2.11$ [22]. The numerical simulated trajectories of phase portraits are plotted in Fig.5. Under the influence of external periodic excitation, the trajectory of phase portraits converts from periodic solution to chaotic attractor. With the stimulus amplitude further increasing, the trajectory of phase portraits converts from chaotic attractor to the period-1behavior. As shown in Fig.6, chaotic attractor is presented for $A = 0.1$, and the period-1 behavior for $A = 0.3$.

In addition to all of the above, let the parameters $\beta$=0.5, $\mu$=0.9, $a$=1, $b$=0.1, $\omega_0 = 1.68$, and system (1) produces coexisting chaotic attractors [22]. Fig.7 (a-b) show that different chaotic attractors can coexist for different initial values (2,0,0,0) and (-2,0,0,0).

When $A = 0.7$, coexisting attractors of the period-3 and the period-1 can be found in system (2) for initial values (2,0,0,0) and (-2,0,0,0). The phase portraits are plotted in Fig.8. It is worth mentioning that the coexisting phenomenon disappears when $A = 0.8$. It shows that coexisting behaviors are affected by the amplitude of periodic excitation in system (2).

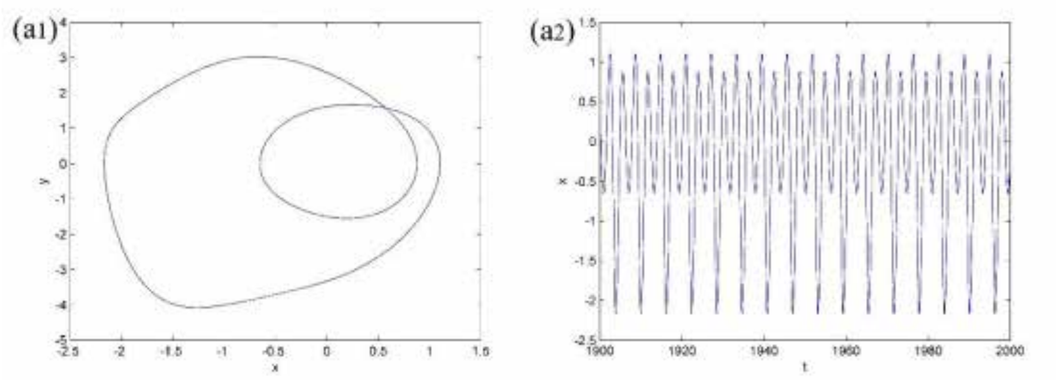


Fig.5. Phase portrait and time series of system (1) with initial conditions (-2,0,0,0) and $\boldsymbol{\omega_0} = \mathbf{2.11}$, (a1) Phase portrait in $\boldsymbol{x} - \boldsymbol{y}$ plane; (a2) Time series of variable $x$.

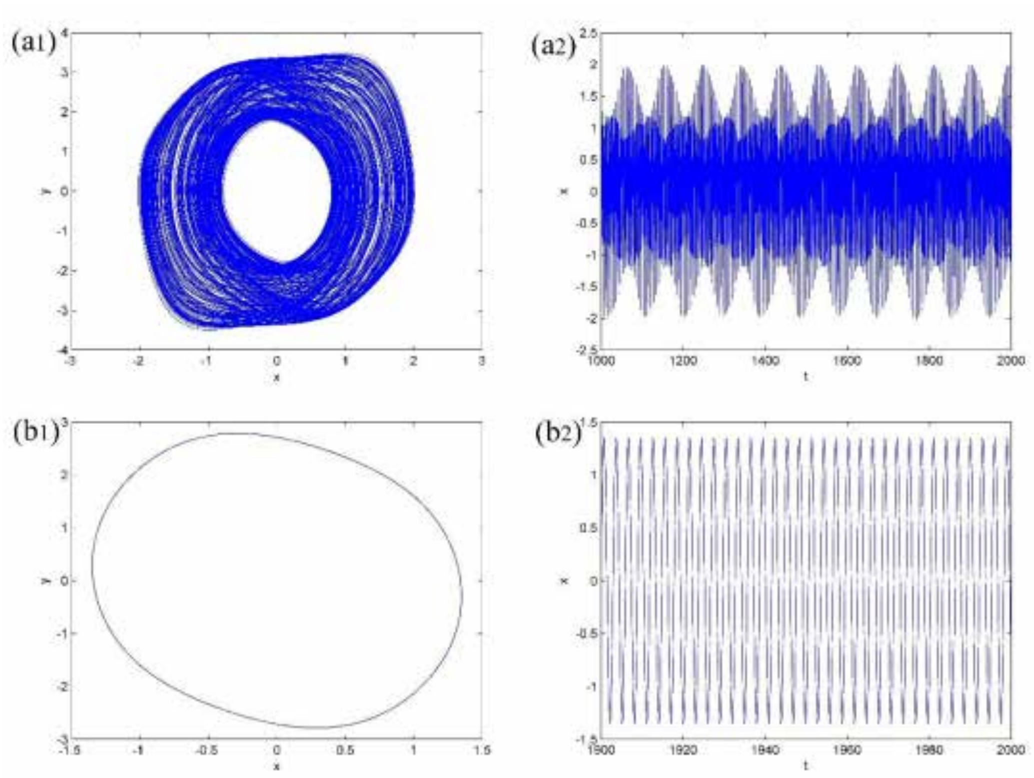


Fig.6. Phase portraits and time series of system (2) with initial value (-2,0,0,0) and $\boldsymbol{\omega_0} = \mathbf{2.11}$. (a1) Phase portrait in $\boldsymbol{x} - \boldsymbol{y}$ plane for $\boldsymbol{A} = \mathbf{0.1}$;(a2) Time series of variable $x$ for $\boldsymbol{A} = \mathbf{0.1}$; (b1) Phase portrait in $\boldsymbol{x} - \boldsymbol{y}$ plane for $\boldsymbol{A} = \mathbf{0.3}$; (b2) Time series of variable $x$ for $\boldsymbol{A} = \mathbf{0.3}$.

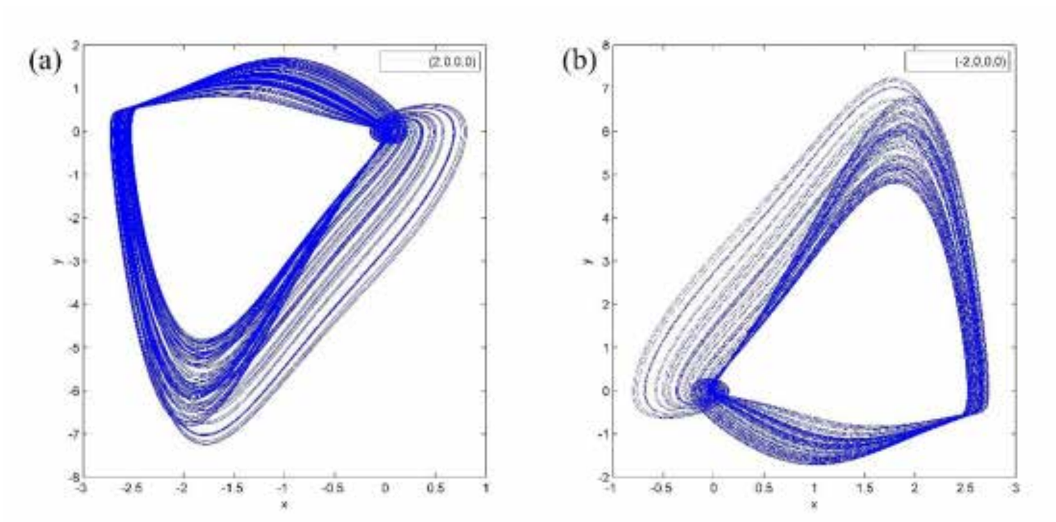


Fig. 7. Coexistence of different chaotic attractors of system (1) with $\boldsymbol{\omega_0 = 1.68}$, (a) Phase portrait in $\boldsymbol{x-y}$ plane for initial value (2,0,0,0); (b) Phase portrait $\boldsymbol{x-y}$ plane for initial value (-2,0,0,0).

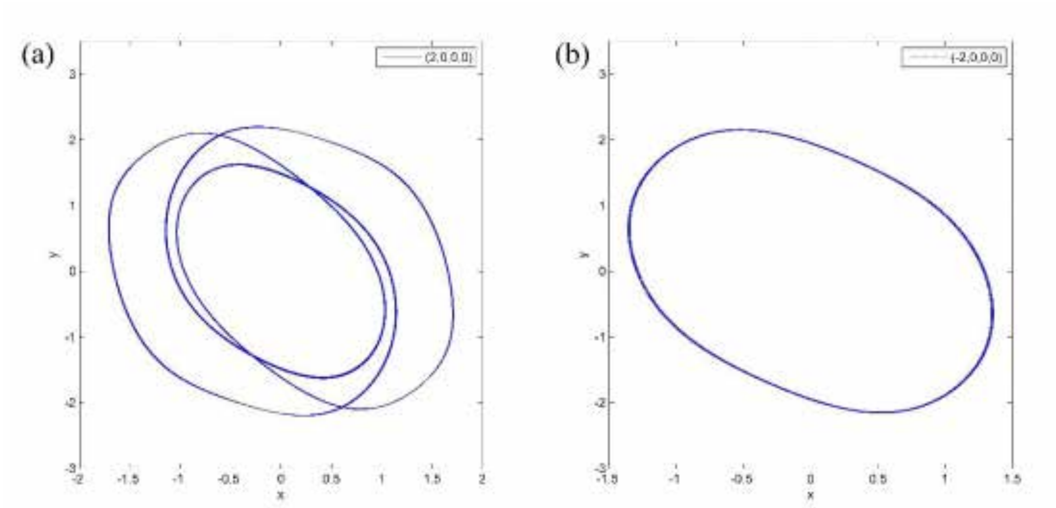


Fig. 8. Coexistence of different attractors of system (2) with $\boldsymbol{\omega_0 = 1.68}$, $\boldsymbol{A = 0.7}$, (a) Phase portrait in $\boldsymbol{x-y}$ plane for initial value (2,0,0,0); (b) Phase portrait in $\boldsymbol{x-y}$ plane for initial value (-2,0,0,0).

## 4. Conclusions

The dynamical behaviors of the system with external periodic excitation are discussed in this paper. When the external periodic excitation is added into the system, the addressed system shows various dynamics with the change of amplitude $A$. When $\omega_0 = 1.68$, the dynamics of the system can convert to period from chaots. When $\omega_0 = 1$, the dynamics of system can convert to bursting oscillations from period. When $\omega_0 = 2.11$, the dynamics of system can convert to chaotic from multicycle. In addition, the coexisting dynamics are also influenced by the amplitude. When $\omega_0 = 1.68$, the original system displays two coexisting chaotic attractors. However, when the external periodic excitation is considered as $A = 0.7$, the system emerges coexisting attractors of period-3 and period-1. Conclusions above may have applications in many fields, such as image encryption, secure communications.

## Acknowledgement

This work is supported by National Natural Science Foundation of China (Grant No. 11472238, No. 51777180) and the Qing Lan Project of the Jiangsu Higher Education Institutions of China.

## References:

1. Edward Norton Lorenz. Deterministic Nonperiodic Flow. Journal of the Atmospheric Sciences, 1963, 20(2):130-141.
2. Guanrong Chen, Tetsushi Ueta. Yet Another Chaotic Attractor. International Journal of Bifurcation and Chaos, 1999, 9(7):1465-1466.
3. Jinhu Lü, Guanrong Chen. A new chaotic attractor coined . International Journal of Bifurcation and Chaos, 2002, 12(3):659-661.
4. Wenbo Liu, Guanrong Chen. A New Chaotic System and its Generation. International Journal of Bifurcation and Chaos, 2003, 13(1):261-267.
5. Xiaodong Luo, Michael Small, Marius-F. Danca, Guanrong Chen. On a dynamical system with multiple chaotic attractors. International Journal of Bifurcation and Chaos, 2007, 17(9):3235-3251.
6. Jinhu Lü, Guanrong Chen, Daizhan Cheng. A new chaotic system and beyond: the generalized Lorenz-like system. International Journal of Bifurcation and Chaos, 2004, 14(5):1507-1537.
7. Gennady A Leonov, Nikolay Vladimirovich Kuznetsov, Timur Mokaev. Hidden attractor and homoclinic orbit in Lorenz-like system describing convective fluid motion in rotating cavity. Communications in Nonlinear Science and Numerical Simulation, 2015, 28(1-3):166-174.
8. Gennady A Leonov, Nikolay Vladimirovich Kuznetsov, Vladimir I. Vagaitsev. Localization of hidden Chua's attractors. Physics Letters A, 2011, 375(23):2230-2233.
9. Bocheng Bao, Pan Jiang, Quan Xu, Mo Chen. Hidden attractors in a practical Chua's circuit based on a modified Chua's diode. Electronics Letters, 2016, 52(1):23-25.
10. Ting Yang. Multistability and Hidden Attractors in a Three-Dimensional Chaotic System. International Journal of Bifurcation and Chaos, 2020, 30(6):17.
11. Sayantan Nag Chowdhury, Dibakar Ghosh. Hidden attractors: A new chaotic system without equilibria. The European Physical Journal Special Topics, 2020, 229(6-7):1299-1308.
12. Zhouchao Wei. Dynamical behaviors of a chaotic system with no equilibria. Physics Letters A, 2011, 376(2):102-108.
13. A. P. Kuznetsov, Sergey Petrovich Kuznetsov, Erik Mosekilde, Nataliya Stankevich. Co-existing hidden attractors in a radio-physical oscillator system. Journal of Physics A: Mathematical and Theoretical, 2015, 48(12):125101.
14. Zhouchao Wei, Rongrong Wang, Anping Liu. A new finding of the existence of hidden hyperchaotic attractors with no equilibria. Mathematics and Computers in Simulation, 2014, 100(1):13-23.
15. Sajad Jafari, J.C. Sprott, S. Mohammad Reza Hashemi Golpayegani. Elementary quadratic chaotic flows with no equilibria. Physics Letters A, 2013, 377(9):699-702.
16. Li Cui, WenHui Luo, QingLi Ou. Analysis and implementation of new fractional-order multi-scroll hidden attractors. Chinese Physics B, 2021, 30(2):238-245.
17. Adel Ouannas, Amina Aicha Khennaoui, Shaher Momani, Viet-Thanh Pham, Reyad El-Khazali. Hidden attractors in a new fractional–order discrete system: Chaos,complexity, entropy, and control. Chinese Physics B, 2020, 29(5):210-217.
18. Dawei Ding, Yecui Weng, Nian Wang. Hidden Attractors in a Delayed Memristive Differential System with Fractional Order and Chaos Synchronization. Journal of Harbin Institute of Technology(New Series), 2020, 27(1):67-75.
19. Yanxia Tang, Abdul Jalil M Khalaf, Karthikeyan Rajagopal, Viet-Thanh Pham, Sajad Jafari, Ye Tian. A new nonlinear oscillator with infinite number of coexisting hidden and self-excited attractors. Chinese Physics B, 2018, 27(4):212-217.
20. Jianghong Bao, Dandan Chen. Coexisting hidden attractors in a 4D segmented disc dynamo with one stable equilibrium or a line equilibrium. Chinese Physics B, 2017, 26(8):18-27.
21. Viet Thanh Pham, Sajad Jafari, Sundarapandian Vaidyanathan, Christos Volos, Xiong Wang. A novel memristive neural network with hidden attractors and its circuitry implementation. Science China Technological Sciences, 2016, 59(3):358-363.
22. Han Bao, Bocheng Bao, Yi Lin, Jiang Wang, Huagan Wu. Hidden attractor and its dynamical characteristic in memristive self-oscillating system. Acta Physica Sinica, 2016, 65(18):219-230.
23. Bocheng Bao, Jianping Xu, Guohua Zhou, Zhenghua Ma, Ling Zou. Chaotic memristive circuit: equivalent circuit realization and dynamical analysis. Chinese Physics B, 2011, 20(12):113-119.
24. Zuolei Wang, Xuerong Shi. Electric activities of time-delay memristive neuron disturbed by Gaussian white noise. Cognitive neurodynamics, 2020, 14(4):115-124.
25. Xuerong Shi, Zuolei Wang. Multiple Firing Patterns in Coupled Hindmarsh-Rose Neurons with a Nonsmooth

Memristor. Neural Plasticity, 2020, 2020:8826369.

26. Bocheng Bao, Yongxin Zhu, Jun Ma, Han Bao, Huagan Wu, Mo Chen. Memristive neuron model with an adapting synapse and its hardware experiments. Science China Technological Sciences, 2021,64(5):1107-1117.
27. Zuolei Wang, Xuerong Shi. Electric activities of time-delay memristive neuron disturbed by Gaussian white noise. Cognitive Neurodynamics, 2020, 14:115-124.
28. Liang Li, Youhua Qian. Numerical Simulation of a Class of Duffing Systems with Periodic Excitation. Dynamical Systems and Control, 2021, 10(1):77-90.
29. Braiman, Goldhirsch. Taming chaotic dynamics with weak periodic perturbations. Physical review letters, 1991, 66(20):2545-2548.
30. Hervé Lucas Koudahoun, Yélomè Judicaël Fernando Kpomahou, Jean Akande, Damien Kêgnidé Kolawolé Adjaï. Chaotic Dynamics of an Extended Duffing Oscillator Under Periodic Excitation. World Journal of Applied Physics, 2018, 3(2):34-50.
31. Jianzhe Huang. Strange Dynamic Behavior Induced by Time Delay for a Rijke Tube System with Periodic Excitation. Journal of Nonlinear Science, 2020, 30(4):767-791.