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Affine Invariant Representation with Generic Polar Radius Integral Transform

Chunyan Liu [1], Jianwei Yang [1,2] ,Chengxi Zhou[3]

[1]School of Mathematics and Statistics, Nanjing University of Information Science and Technology, Nanjing, China, 210044

[2]Guangdong Key Laboratory of Intelligent Information Processing and Shenzhen Key Laboratory of Media Security, Shenzhen , China, 518060

[3]Jiayuan Technology Co., Ltd, Nanjing, China, 210019

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Abstract: In many computer vision tasks, the extraction of features invariant to affine transform plays an important role. To achieve better accuracy, region-based approaches usually need expensive computation. Whereas, contour-based methods need less computation, but their performance is strongly dependant on the boundary extraction. A method, *generic polar radius integral transform* (GPRIT), is proposed to combine region-based and contour-based method together for the extraction of affine invariant features. Polar radius integral transform and central projection transform are all special cases of the proposed GPRIT. With GPRIT, any object is converted into a closed curve for data reduction. Consequently, stationary wavelet transform is conducted to construct affine invariants. Several experiments have been presented to evaluate performance of the proposed GPRIT.

Keywords: *generic polar radius integral transform* (GPRIT), invariant, affine transform, feature extraction.

1. Introduction

Image is an important communication tool and information medium in daily production and life. Image feature extraction is one of the key technologies in computer vision and pattern recognition. Affine transform, including rotation, translation, scaling and shearing transformations[1][2], can be used as the approximate model for images of the same object from different viewpoints. Affine invariant features have been applied to target recognition[3][4][5][6], image registration[7][8], digital watermarking[9][10] and many other fields. Hence, the study of affine invariant feature extraction has attracted wide attention[11][12].

To extract affine invariant features, a great number of methods have been developed. Based on whether invariant features are extracted from the contour only or from the whole shape region, these methods can be divided into categories:[13][14][15][16] contour-based and region-based. Both of these two types of methods have their merits and shortcomes.

Contour-based techniques employ boundary of objects for the extraction of invariant features, and they are often of better data reduction. Fourier descriptor[17][18] and wavelet transform[19][20][21] are two widely utilized contour-based technique. But contour-based methods are strongly dependant on the extraction of contours. They are usually invalid to objects which are consisting of several components. Consequently, applications of contour-based methods are limited.

In contrast to contour-based techniques, region-based approaches usually get high accuracy, but some of these approaches are of high computational demands. *Affine moment invariants* (AMIs)[22][23] are the most famous region-based method. But AMIs are sensitive to noise. To improve robustness of moment-based techniques to noise, *cross-weighted moment* (CWM)[24] and *multi-scale autoconvolution* (MSA)[25] have been put forward. But the computational cost of these methods are extremely expensive.

In this paper, *generic polar radius integral transform* (GPRIT) is proposed to combine contour-based technique with region-based technique. By GPRIT, any object is converted into a closed curve. All pixels in the image have been utilized. Then, parameterization and stationary wavelet transform are conducted on the obtained closed curve. The utilized technique is contour-based.

Recently, *central projection transform* (CPT)[26] and *polar radius integral transform* (PRIT)[27] have been proposed to combine region-based and contour-based techniques together. They are only special cases of the proposed GPRIT. Experiments have also been conducted to demonstrate performance of the proposed GPRIT. Results show that the derived features are invariant to affine transform. Furthermore, GPRIT with small s is more robust to noise.

The rest of this paper is organized as follows: In section 2, the definition of GPRIT is provided. The affine invariance is also discussed. Algorithm is developed for the extracting of affine invariant features in section 3. Experimental results are presented in section 4. Finally, some conclusion remarks are given in section 5.

2. GPRIT and its affine invariance

The definition of GPRIT is provided. Affine invariance of GPRIT is also discussed.

2.1. Definition of GPRIT

To conduct GPRIT on an image $I(x,y)$, the Cartesian coordinate system needs to be transformed to polar coordinate system. The origin is firstly translated to $O(x_0, y_0)$, the centroid of image $I(x,y)$. Here,

$$x_0 = \frac{\iint xI(x,y)dxdy}{\iint I(x,y)dxdy}, \qquad y_0 = \frac{\iint yI(x,y)dxdy}{\iint I(x,y)dxdy}.$$

In polar coordinate system, $f(r,\theta)$ is utilized to denote image $I(x,y)$.

Definition 1. *For $s \ge 0$ and $h, t \in R$, the generic polar radius integral transform (GPRIT) of image $f(r,\theta)$ is defined as follows:*

$$g_{s,h}^{t}(\theta) = \frac{(\int r^s f(r,\theta)dr)^h}{(\int f(r,\theta)dr)^t}. \tag{1}$$

By Eq .(1), it can be observed that $g_{s,h}^{t}(\theta)$ is a single-valued function. The set $\{g_{s,h}^{t}(\theta)\cos\theta, g_{s,h}^{t}(\theta)\sin\theta) \mid \theta \in [0,2\pi)\}$ forms a closed curve in Cartesian coordinate system. As a result, GPRIT converts any object into a closed curve $g_{s,h}^{t}(\theta)$. For example, Fig. 1(a) is an image of Coil-20 and Fig. 1(b) is the GPRIT of Fig. 1(a). It is a closed curve. In this paper, GPRIT is employed to extract affine invariant features.

Remark 1. *GPRIT is the generalization of PRIT and CPT.*

In fact, if we set $s = t = 0$ and $h = 1$, then $g_{s,h}^{t}(\theta)$ in Eq .(1) is the same as CPT defined in [26]. If we set $t = 0$, then $g_{s,h}^{t}(\theta)$ in Eq .(1) is the same as PRIT defined in [27].

2.2. Affine invariance of GPRIT

Affine transform is the transformation defined as follows

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix},$$

where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a non-singular matrix. For perspective distortions, affine transform can be utilized as the best linear approximation model [1][2]. It includes not only similarity transform (translation, rotation and scaling), but also shearing. The following theorem shows that the proposed GPRIT keeps the affine transform relation.

Theorem 1. *For $s \ge 0$ and $h \in R$, let $t = h(s+1) - 1$, $\tilde{f}(\tilde{r}, \tilde{\theta})$ denotes the affine transformed image of $f(r,\theta)$. Let $\tilde{g}_{s,h}^{t}(\tilde{\theta})$ be GPRIT of $\tilde{f}(\tilde{r}, \tilde{\theta})$, and $g_{s,h}^{t}(\theta)$ be GPRIT of $f(r,\theta)$. Then the following relations hold*