Journal of Nonlinear Modeling and Analysis https://global-sci.com/jnma
Volume 8, Number 2, April 2026, 570–586 DOI:10.12150/jnma.2026.570

Solving the Kuramoto-Sivashinsky Equation via a Modified Adomian Decomposition Method

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Received 19 March 2025; Accepted 23 August 2025

Abstract The Kuramoto-Sivashinsky equation (KSE) is a well-known nonlinear partial differential equation (PDE) that plays a significant role in various scientific fields, particularly in fluid dynamics and reaction-diffusion systems. In this study, we employ the Modified Adomian Decomposition Method (MADM) to derive an analytical solution to the KSE over the domain $t \in \mathbb{R}$ and $0 \leq x < 1$. The research provides a comprehensive analysis of the applicability of MADM in generating explicit series solutions for the KSE. The equation is formulated under proper initial conditions, and the systematic implementation of MADM is demonstrated involving the decomposition of nonlinear terms and successive approximation. The convergence and stability of the resulting series solutions are investigated, highlighting the method's efficiency in capturing the dynamics described by the KSE. Numerical simulations are presented to validate the analytical results, illustrating the effectiveness of the MADM in solving the KSE within the given parameters. This study not only enhances the theoretical understanding of the KSE but also serves as a practical guide for applying analytical techniques to complex nonlinear PDEs. Moreover, the proposed method represents a significant advancement in solving highly non-linear models by providing a straightforward recursive scheme that avoids linearization or small-parameter assumptions. The physical relevance of the obtained solutions is reflected in their ability to capture key features of instability and chaotic behavior characteristic of the KSE. These findings underscore the potential of MADM as a powerful and versatile tool in the study of spatiotemporal phenomena governed by nonlinear PDEs.

Keywords Non-linear evolution equations, decomposition techniques, series solutions, initial value problems

MSC(2010) 35Q53, 35c10, 35A25

1. Introduction

The KSE is a prominent fourth-order nonlinear partial differential equation that arises in various fields, including fluid dynamics, combustion theory, and pattern formation. Its complexity and its role in capturing chaotic behavior make it a critical subject of study in applied mathematics and physics. This equation effectively models processes characterized by instability and turbulence. For example, Hyman and

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Nicolaenko [12] demonstrated how the KSE bridges partial differential equations and dynamical systems to capture chaotic and complex spatiotemporal behavior. Cousin and Larkin [5] analyzed the KSE in domains with moving boundaries, highlighting its role in dynamic boundary interactions. Ebiwareme [8] empolyed the tanh-coth method to obtain exact and solitary solutions, illustrating wave propagation and nonlinear wave behavior. Khater and Temsah [14] applied Chebyshev spectral collocation methods to compute accurate numerical solutions of the generalized KSE. Lu et al. [23] proposed data-driven stochastic model reduction approaches for the KSE, supporting its utility in uncertainty quantification and simplified modeling. Shah et al. [27] introduced semi-analytical methods to solve families of KSEs, offering insights into efficient computational handling. Troy [29] addressed the existence of steady-state solutions to the KSE, which is crucial for understanding long-term system behavior. Zavalani [31] implemented exponential time differencing methods to enhance temporal integration accuracy, while Zhonghua et al. [32] developed fully discrete Galerkin methods and provided rigorous error estimates contributing to numerical stability and simulation precision. Recently, various analytical methods have been effectively utilized in combination with decomposition techniques to address a wide range of linear and nonlinear initial and boundary value problems. For instance, recent studies have employed efficient analytical frameworks such as the Aboodh transformation and Laplace based decomposition methods to tackle integro-differential systems and fractional dynamical equations without the need for discretization or perturbation [15, 16]. Moreover, researchers have introduced significant modifications to classical decomposition techniques to enhance their flexibility and accuracy when applied to nonlinear systems [17–19]. These developments include iterative procedures and polynomial-based schemes that improve convergence and reduce computational cost [20, 21]. By incorporating such advancements, this study aligns with recent analytical trends and contributes to the growing body of literature focused on the enhancement of decomposition-based methods for complex differential systems.

Here, we implement the MADM to address the KSE over the domain $t \in \mathbb{R}$ and $0 \leq x < 1$.

We investigate the dynamic behavior of the KSE in a bounded spatial setting, emphasizing the mathematical challenges associated with its nonlinear and higher-order derivatives. The spatial domain is chosen to explore localized phenomena and boundary effects that are significant in many physical applications. Several analytical methods have been developed to study this equation, each addressing different aspects of its solutions and properties. These include: Homotopy Analysis Method (HAM) [6], $\frac{1}{G'}$ expansion method [30] and Adomian Decomposition Method [24].

The Adomian Decomposition Method (ADM) [1] is a powerful and versatile method used for solving nonlinear partial differential equations (PDEs). This technique breaks down the problem into a series of decomposed equations that are easier to solve, making it particularly effective for complex nonlinear systems. It represents the solution as an infinite series, computed recursively through the following key steps [2]:

1.**Function decomposition**: The nonlinear differential equation is expressed in terms of a linear operator and a nonlinear operator. The solution $u(x,t)$ is decomposed into a series, typically as follows:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \dots,$$

where u_0 is the initial approximation (often the solution of the linear part), and u_n represents the successive corrections.

2.**Application of operators**: The linear and nonlinear components of the differential equation are identified. The nonlinear part is usually dealt with using Adomian polynomials, which express the nonlinear function in terms of a series expansion, generating a sequence of polynomials A_n that can be computed recursively.

3.**Recursive formulation**: After setting the initial condition, the method recursively computes each term of the series:

$$u_{n+1} = L^{-1}(N(u_0 + u_1 + \ldots + u_n)),$$

where L^{-1} is the inverse of the linear operator and N is the nonlinear operator.

4.**Convergence**: The series solution is typically truncated after a finite number of terms for practical purposes. Under certain conditions, the series converges to the exact solution of the nonlinear PDE. Many studies have used the ADM to solve different differential equations such as [4, 7, 9–11, 13, 26, 28].

Here, we implement the MADM to solve the KSE over the domain $t \in \mathbb{R}$ and $0 \leq x < 1$. The MADM enhances the standard Adomian method by modifying the decomposition process to effectively handle the nonlinear terms, allowing for the generation of a series solution that converges to the true solution of the KSE. This manuscript ditails the MADM procedure, applies it to the KSE, and discusses the resulting solutions in terms of stability and dynamic behavior within the defined spatial domain. This work is expected to contribute substantially to the existing literature on analytical solutions of the KSE, providing insights into the nature of its solutions and their relevance to practical applications.

Ultimately, the study focuses on applying MADM to the KSE and examining the uniqueness and behavior of its solution particularly given the equation's intrinsic nonlinearity and complexity. The study is localized within the spatial domain defined as $t \in \mathbb{R}$ and $0 \leq x < 1$, which allows for a detailed examination of the equation's behavior.

2. Modified adomian decomposition method

In this study, we highlight a key modification in the decomposition method applied to the following equation [6, 24, 25]:

$$u_t + uu_x + \alpha u_{xx} + \gamma u_{xxx} + \beta u_{xxxx} = 0, \tag{2.1}$$

where $t \in \mathbb{R}$ and $x \in [0, 1)$, subject to the initial conditions:

$$u(0,t) = f_0(t), \quad u_x(0,t) = f_1(t), \quad u_{xx}(0,t) = f_2(t), \quad u_{xxx}(0,t) = f_3(t). \tag{2.2}$$

Here, α, γ, and β are real constants.

In the classical ADM, the time derivative u_t is typically chosen as the linear operator, while all spatial derivatives and nonlinear terms are treated as source terms. The nonlinear terms are then decomposed using the standard Adomian polynomials.

However, in our proposed Modified Adomian Decomposition Method (MADM), the linear differential operator is redefined to include the second- and fourth-order spatial derivatives u_{xx} and u_{xxxx}, whereas the time derivative u_t and the nonlinear term uu_x remain in the source term.

It is important to emphasize that the Adomian polynomials used to decompose the nonlinear term are exactly the same as in the classical ADM, and no modifications were introduced in their formulation.

This redefinition of the linear operator improves the convergence behavior and accuracy of the approximate solution, especially for problems dominated by higher-order spatial derivatives. The modification is particularly effective in the domain $t \in \mathbb{R}, x \in [0,1)$, which is commonly encountered in many physical and engineering applications, where α, γ and β are real Constants. In previous analytical studies such as [6, 24], the KSE was examined over the domain ($x \in [0, 32\pi]$ and $t \in [0,1)$ and, with initial condition. In contrast, our study considers the domain and, allowing for the investigation of long-time dynamics in a spatially localized setting. This domain shift introduces new analytical challenges and offers novel insights into the solution behavior under different initial conditions. We can write equation (2.1) as:

$$u_{xxxx} + \frac{\alpha}{\beta} u_{xx} = -\frac{1}{\beta} u_t - \frac{1}{\beta} uu_x - \frac{\gamma}{\beta} u_{xxx},$$

this is equivalent to:

$$L_x(u) = -\frac{1}{\beta} u_t - \frac{1}{\beta} uu_x - \frac{\gamma}{\beta} u_{xxx}, \tag{2.3}$$

where L_x defined is by:

$$L_x(u) = e^{(-\sqrt{-\frac{\alpha}{\beta}})x} \frac{\partial}{\partial x} e^{(\sqrt{-\frac{\alpha}{\beta}})x} \frac{\partial^2}{\partial x^2} e^{(\sqrt{-\frac{\alpha}{\beta}})x} \frac{\partial}{\partial x} e^{(-\sqrt{-\frac{\alpha}{\beta}})x}(u). \tag{2.4}$$

The inverse operator L_x^{-1} is given by:

$$L_x^{-1}(u) = e^{(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x e^{(\sqrt{-\frac{\alpha}{\beta}})x}(u) dx dx dx dx. \tag{2.5}$$

Applying L_x^{-1} to both sides of equation (2.3), we obtain:

$$u(x,t) = \aleph(x,t) - L_x^{-1}\frac{1}{\beta} u_t(x,t) - L_x^{-1}\frac{1}{\beta} u(x,t)u_x(x,t) - L_x^{-1}\frac{\gamma}{\beta} u_{xxx}(x,t). \tag{2.6}$$

We define the nonlinear term as $N(x,t,u,u_x) = uu_x$. The nonlinear term is decomposed using the Standard Adomian polynomials from ADM. Based on Standard Adomian Method, the solution is expressed as a series [3]:

$$u_0(x,t) = \aleph(x,t),$$

and

$$u_{m+1} = -\frac{1}{\beta} L_x^{-1} u_{mt}(x,t) - \frac{\gamma}{\beta} L_x^{-1} u_{mxxx} - \frac{1}{\beta} L_x^{-1} A_m, \qquad m = 0, 1, 2, 3, \ldots,$$

where A_m are the Adomian polynomials defined by:

$$A_m = \frac{1}{m!} \frac{d^m}{d\lambda^m} \left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right) \right]_{\lambda=0}, \qquad m = 0, 1, 2, 3, \ldots\ .$$

The first few Adomian polynomials are

$$A_0 = u_0 u_{0x},$$

$$A_1 = u_0 u_{1x} + u_1 u_{0x},$$

$$A_2 = u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x},$$

and so on.

3. Main result

This section presents existence and uniqueness results of Eq.(2.1) with initial conditions specified in Eq.(2.2) and provides a proof of them. Before delving into the main results, the following hypotheses is introduced:
(A1) There exist two constants $\mathfrak{C}_1, \mathfrak{C}_2 > 0$ such that for $u, v \in C(J, R)$, and let $C(J, R)$ be a Banach space of all continuous real value function with norm $\|u\|_\infty = sup|u(x, t|; (x, t) \in J$.

$$|N(x, t, u, u_x) - N(x, t, v, v_x)| \leq \mathfrak{C}_1 |u - v| + \mathfrak{C}_2 |u_x - v_x|.$$

(A2) There exist two constants $\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 > 0$ such that

$$|u_t - v_t| \leq \mathfrak{E}_1 |u - v|,$$

$$|u_x - v_x| \leq \mathfrak{E}_2 |u - v|,$$

$$|u_{xxx} - v_{xxx}| \leq \mathfrak{E}_3 |u - v|.$$

3.1. Existence and uniqueness

Theorem 3.1. *Assume that (A1) and (A2) are true if*

$$\mathfrak{L} = \frac{(\mathfrak{E}_1 + \mathfrak{E}_3 + \mathfrak{C}_1 + \mathfrak{C}_2 \mathfrak{E}_2) \left(\mathfrak{b}^2 \alpha - 2\beta + 2\beta \cos \left(\frac{\mathfrak{b}\sqrt{\alpha}}{\sqrt{\beta}} \right) \right) \beta}{2\alpha^2} < 1.$$

Thus, there is a unique solution $u(x, t) \in C(J, R)$ to (1)-(2).

Proof.

$$\begin{aligned} u(x,t) =& \aleph(x,t) - e^{(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x \\ & e^{(\sqrt{-\frac{\alpha}{\beta}})x} \left(-\frac{1}{\beta} u_t(x,t) - \frac{\gamma}{\beta} u_{xxx}(x,t)\right) dxdxdxdx \\ & - e^{(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x e^{(\sqrt{-\frac{\alpha}{\beta}})x} \\ & N(x,t,u,u_x) dxdxdxdx. \end{aligned}$$

Let $\mathfrak{D} : C(J, R) \to C(J, R)$ be defined by

$$(\mathfrak{D}u)(x,t) = \aleph(x,t) - e^{(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x \int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x} \int_0^x$$

$$e^{(\sqrt{-\frac{\alpha}{\beta}})x}(-\frac{1}{\beta}u_t(x,t)-\frac{\gamma}{\beta}u_{xxx}(x,t))dxdxdxdx$$
$$-e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{(\sqrt{-\frac{\alpha}{\beta}})x}$$
$$N(x,t,u,u_x)dxdxdxdx.$$

If $u(x,t)\in C(J,R)$ is a fixed point of $\mathfrak{D}$, we can conclude that u satisfies equations (1)-(2). It has been demonstrated that $\mathfrak{D}$ has a fixed point u in $C(J,R)$. Let $u,v\in C(J,R)$ and consider any value of x such that $0<x\leq\mathfrak{b}$

$$\begin{aligned}u(x,t)=&\aleph(x,t)-e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\\&e^{(\sqrt{-\frac{\alpha}{\beta}})x}(\frac{1}{\beta}u_t(x,t)+\frac{\gamma}{\beta}u_{xxx}(x,t))dxdxdxdx\\&-e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{(\sqrt{-\frac{\alpha}{\beta}})x}\\&N(x,t,u,u_x)dxdxdxdx,\end{aligned}$$

and

$$\begin{aligned}v(x,t)=&\aleph(x,t)-e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\\&e^{(\sqrt{-\frac{\alpha}{\beta}})x}(\frac{1}{\beta}v_t(x,t)+\frac{\gamma}{\beta}v_{xxx}(x,t))dxdxdxdx\\&-e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{(\sqrt{-\frac{\alpha}{\beta}})x}\\&N(x,t,v,v_x)dxdxdxdx.\end{aligned}$$

We get

$$\begin{aligned}&|(\mathfrak{D}u)(x,t)-(\mathfrak{D}v)(x,t)|\\=&e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\\&e^{(\sqrt{-\frac{\alpha}{\beta}})x}(\frac{1}{\beta}((v_t(x,t)-u_t(x,t))+\frac{\gamma}{\beta}(v_{xxx}-u_{xxx}(x,t)))dxdxdxdx\\&-e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\\&e^{(\sqrt{-\frac{\alpha}{\beta}})x}(N(x,t,v,v_x)-N(x,t,u,u_x))dxdxdxdx,\\\leq&e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\\&e^{(\sqrt{-\frac{\alpha}{\beta}})x}(\mathfrak{E}_1|u-v|+\mathfrak{E}_2|u-v|+\mathfrak{E}_3|u-v|+\mathfrak{C}_1\mathfrak{C}_2|u-v|)dxdxdxdx\\=&e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\\&e^{(\sqrt{-\frac{\alpha}{\beta}})x}(\mathfrak{E}_1+\mathfrak{E}_2+\mathfrak{E}_3+\mathfrak{C}_1\mathfrak{C}_2)|u-v|dxdxdxdx\\=&(\mathfrak{E}_1+\mathfrak{E}_2+\mathfrak{E}_3+\mathfrak{C}_1\mathfrak{C}_2)|u-v|e^{(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\int_0^x\int_0^x e^{-(\sqrt{-\frac{\alpha}{\beta}})x}\end{aligned}$$

$$\int_0^x e^{(\sqrt{-\frac{\alpha}{\beta}})x} dxdxdxdx,$$

that is

$$\begin{aligned}
&|(\mathfrak{D}u)(x,t) - (\mathfrak{D}v)(x,t)| \\
\leq &\frac{(\mathfrak{E}_1 + \mathfrak{E}_3 + \mathfrak{C}_1 + \mathfrak{C}_2\mathfrak{E}_2)\left(\mathfrak{b}^2\alpha - 2\beta + 2\beta\cos\left(\frac{\mathfrak{b}\sqrt{\alpha}}{\sqrt{\beta}}\right)\right)\beta}{2\alpha^2}|u-v|, \\
&|(\mathfrak{D}u)(x,t) - (\mathfrak{D}v)(x,t)| \leq \mathfrak{L}|u-v|.
\end{aligned}$$

By taking sup to both sides we get

$$||(\mathfrak{D}u)(x,t) - (\mathfrak{D}v)(x,t)||_\infty \leq \mathfrak{L}||u-v||_\infty.$$

This indicates that $\mathfrak{D}$ is a contraction mapping. According to the Banach contraction principle, we can conclude that $\mathfrak{D}$ has a unique fixed point $u \in C(J, R)$. □

3.2. Stability

Theorem 3.2. *Let $u(x,t)$ be a solution to the integral equation obtained via the inverse operator L_x^{-1} as follows:*

$$\begin{aligned}
u(x,t) = \mathcal{F}(x,t) - L_x^{-1}\left(\frac{1}{\beta}u_t(x,t)\right) - L_x^{-1}\left(\frac{1}{\beta}u(x,t)u_x(x,t)\right) \\
- L_x^{-1}\left(\frac{\gamma}{\beta}u_{xxx}(x,t)\right).
\end{aligned}$$

Then the method is stable under small perturbations in the initial data.

Proof. Let $u(x,t)$ and $v(x,t)$ be two approximate solutions corresponding to slightly different initial conditions. Define $w(x,t) = u(x,t) - v(x,t)$. Then we get:

$$\begin{aligned}
|w(x,t)| &= \left|L_x^{-1}\left[\frac{1}{\beta}(u_t - v_t) + \frac{1}{\beta}(uu_x - vv_x) + \frac{\gamma}{\beta}(u_{xxx} - v_{xxx})\right]\right| \\
&\leq L_x^{-1}\left[\frac{1}{\beta}|u_t - v_t| + \frac{1}{\beta}|uu_x - vv_x| + \frac{\gamma}{\beta}|u_{xxx} - v_{xxx}|\right].
\end{aligned}$$

Using the hypotheses (A1) and (A2), we conclude that:

$$\|w\| \leq \mathfrak{L}\|u-v\|,$$

for some constant $\mathfrak{L} < 1$, which proves stability. □

4. Convergence

Theorem 4.1. *Assume that the MADM series solution is given by:*

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t),$$

where each u_n is computed recursively. Then the series converges uniformly to the exact solution if the Lipschitz condition is satisfied and $L < 1$, where L is the contraction constant.

Proof. Let the sequence $\{u_n\}$ be generated by the recurrence relation defined by the MADM. We show that $\{u_n\}$ is a Cauchy sequence in the Banach space $C(J, \mathbb{R})$.

$$\|u_{n+1} - u_n\| \leq L\|u_n - u_{n-1}\|,$$

for $0 < L < 1$. By recursion,

$$\|u_{n+1} - u_n\| \leq L^n\|u_1 - u_0\|,$$

which implies that the series $\sum u_n$ converges uniformly. By the completeness of the Banach space, the series converges to a unique fixed point $u(x,t)$ satisfying the integral equation. $\square$

5. Application

Example 5.1. We solve Eq.(2.1) when $\beta = \alpha = 1$, and $\gamma = 0$ with initial conditions (2.2) [6, 22, 24], by applying the steps in Section 2. The components u_0, u_1, u_2, u_3 are

$$\begin{aligned}
u_0 =& (\cos(t) - \cos(t)\sin(t)) + x\left(\frac{\cos^2(t)}{16} + \frac{\sin(t)}{16} - \frac{\sin^2(t)}{16}\right) \\
&+ x^2\left(-\frac{\cos(t)}{512} + \frac{1}{128}\cos(t)\sin(t)\right) + x^3\left(-\frac{\cos^2(t)}{6144} - \frac{\sin(t)}{24576} + \frac{\sin^2(t)}{6144}\right) \\
&+ x^4\left(\frac{\cos(t)}{6144} - \frac{\cos(t)\sin(t)}{1536}\right) + x^5\left(\frac{\cos^2(t)}{122880} + \frac{\sin(t)}{491520} - \frac{\sin^2(t)}{122880}\right) \\
&+ x^6\left(-\frac{\cos(t)}{184320} + \frac{\cos(t)\sin(t)}{46080}\right) + x^7\left(-\frac{\cos^2(t)}{5160960} - \frac{\sin(t)}{20643840} + \frac{\sin^2(t)}{5160960}\right) \\
&+ x^8\left(\frac{\cos(t)}{10321920} - \frac{\cos(t)\sin(t)}{2580480}\right) + x^9\left(\frac{\cos^2(t)}{371589120} + \frac{\sin(t)}{1486356480} - \frac{\sin^2(t)}{371589120}\right) \\
&+ x^{10}\left(-\frac{\cos(t)}{928972800} + \frac{\cos(t)\sin(t)}{232243200}\right),
\end{aligned}$$

$$\begin{aligned}
u_1 =& x^4\left(-\frac{\cos(t)}{1536} + \frac{1}{24}\cos(2t) - \frac{1}{512}\cos(3t) + \frac{\sin(t)}{24} - \frac{1}{384}\cos(t)\sin(t) + \frac{\sin(4t)}{1536}\right) \\
&+ x^5\left(-\frac{\cos(t)}{1920} + \frac{\cos(2t)}{30720} - \frac{\cos(4t)}{30720} - \frac{\sin(t)}{122880} + \frac{67\cos(t)\sin(t)}{30720} - \frac{\sin(2t)}{20480} - \frac{3\sin(3t)}{40960}\right) \\
&+ x^6\left(257\frac{\cos(t)}{11796480} - \frac{13\cos(2t)}{9216} + \frac{53\cos(3t)}{786432} - \frac{257\sin(t)}{184320} + \frac{18241\cos(t)\sin(t)}{188743680}\right. \\
&\left.- \frac{1601\sin(2t)}{377487360} - \frac{17\sin(4t)}{737280}\right) \\
&+ x^7\left(-\frac{169}{220200960} + \frac{257\cos(t)}{20643840} - \frac{155\cos(2t)}{132120576} + \frac{199\cos(4t)}{165150720} + \frac{19\sin(t)}{16515072}\right. \\
&\left.- \frac{13\cos(t)\sin(t)}{258048} + \frac{19\sin(3t)}{6881280}\right) \\
&+ x^8\left(-\frac{2711\cos(t)}{7046430720} + \frac{11\cos(2t)}{430080} - \frac{2967\cos(3t)}{2348810240} + \frac{43\sin(t)}{1720320}\right.
\end{aligned}$$

$$
\begin{aligned}
&-\frac{18241\cos(t)\sin(t)}{10569646080}+\frac{347\sin(2t)}{6039797760}+\frac{113\sin(4t)}{251658240}\Big)\\
&+x^9\left(\frac{5041}{225485783040}-\frac{43\cos(t)}{247726080}+\frac{44887\cos(2t)}{2029372047360}-\frac{433\cos(4t)}{18119393280}\right.\\
&\left.-\frac{1525\sin(t)}{50734301184}+\frac{11\cos(t)\sin(t)}{15482880}-\frac{4541\sin(3t)}{84557168640}\right)\\
&+x^{10}\left(\frac{7937\cos(t)}{1902536294400}-\frac{67\cos(2t)}{232243200}+\frac{9473\cos(3t)}{634178764800}-\frac{37\sin(t)}{132710400}\right.\\
&\left.+\frac{18241\cos(t)\sin(t)}{951268147200}-\frac{17\sin(2t)}{46976204800}-\frac{2657\sin(4t)}{475634073600}\right),
\end{aligned}
$$

$$
\begin{aligned}
u_2=&x^7\left(-\frac{\cos(t)}{20160}+\frac{\cos(2t)}{143360}-\frac{\cos(3t)}{6720}+\frac{\cos(4t)}{161280}-\frac{\cos(6t)}{1290240}\right.\\
&\left.+\frac{\sin(t)}{215040}-\frac{\sin(2t)}{10080}+\frac{\sin(3t)}{1290240}+\frac{\sin(4t)}{20160}-\frac{\sin(5t)}{258048}\right),
\end{aligned}
$$

$$
\begin{aligned}
u_3=&x^{10}\left(-\frac{11\cos(t)}{3715891200}+\frac{\cos(2t)}{6451200}-\frac{\cos(3t)}{82575360}+\frac{\cos(4t)}{7257600}-\frac{\cos(5t)}{247726080}-\frac{\cos(6t)}{58060800}\right.\\
&+\frac{\cos(7t)}{530841600}+\frac{\sin(t)}{9676800}-\frac{5604930907\cos(t)\sin(t)}{121762322841600}+\frac{53366563\sin(2t)}{2319282339840}\\
&\left.+\frac{\sin(3t)}{58060800}+\frac{\sin(4t)}{206438400}-\frac{\sin(5t)}{11612160}+\frac{\sin(6t)}{206438400}-\frac{\sin(8t)}{3715891200}\right).
\end{aligned}
$$

MADM solution is given by

$$
\begin{aligned}
u(x,t)=&u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots+u_{10}(x,t)\\
=&\left(\cos(t)-\frac{1}{2}\sin(2t)\right)+x\left(\frac{1}{16}\cos(2t)+\frac{1}{16}\sin(t)\right)\\
&+x^2\left(-\frac{1}{512}\cos(t)+\frac{1}{256}\sin(2t)\right)\\
&+x^3\left(-\frac{1}{6144}\cos(2t)-\frac{1}{24576}\sin(t)\right)\\
&+x^4\left(-\frac{1}{2048}\cos(t)+\frac{1}{24}\cos(2t)-\frac{1}{512}\cos(3t)+\frac{1}{24}\sin(t)\right.\\
&\left.-\frac{5}{3072}\sin(2t)+\frac{1}{1536}\sin(4t)\right)+x^5\left(-\frac{1}{1920}\cos(t)+\frac{1}{24576}\cos(2t)\right.\\
&\left.-\frac{1}{30720}\cos(4t)-\frac{1}{163840}\sin(t)+\frac{1}{960}\sin(2t)-\frac{3}{40960}\sin(3t)\right)\\
&+x^6\left(\frac{193}{11796480}\cos(t)-\frac{13}{9216}\cos(2t)+\frac{53}{786432}\cos(3t)-\frac{257}{184320}\sin(t)\right.\\
&\left.+\frac{9}{163840}\sin(2t)-\frac{17}{737280}\sin(4t)\right)+x^7\left(-\frac{169}{220200960}-\frac{767}{20643840}\cos(t)\right.\\
&+\frac{247}{44040192}\cos(2t)-\frac{1}{6720}\cos(3t)+\frac{1223}{165150720}\cos(4t)-\frac{1}{1290240}\cos(6t)\\
&+\frac{95}{16515072}\sin(t)-\frac{107}{860160}\sin(2t)+\frac{73}{20643840}\sin(3t)+\frac{1}{20160}\sin(4t)\\
&\left.-\frac{1}{258048}\sin(5t)\right)+x^8\left(-\frac{1}{161280}-\frac{58333}{2348810240}\cos(t)+\frac{41}{1290240}\cos(2t)\right.
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{5827}{4227858432}\cos(3t)-\frac{1}{161280}\cos(4t)+\frac{19}{55050240}\cos(5t)+\frac{157}{5160960}\sin(t)\\
&+\frac{684457}{14092861440}\sin(2t)-\frac{17}{1290240}\sin(3t)+\frac{1559}{1761607680}\sin(4t)\\
&-\frac{1}{11796480}\sin(6t)\Big)+x^9\left(\frac{50489}{2029372047360}+\frac{827}{743178240}\cos(t)\right.\\
&-\frac{38893}{45097156608}\cos(2t)+\frac{283}{61931520}\cos(3t)-\frac{9053}{42278584320}\cos(4t)\\
&+\frac{53}{1981808640}\cos(6t)-\frac{16949}{50734301184}\sin(t)+\frac{151}{46448640}\sin(2t)\\
&\left.-\frac{6493}{84557168640}\sin(3t)-\frac{11}{6635520}\sin(4t)+\frac{199}{1585446912}\sin(5t)\right)\\
&+x^{10}\left(\frac{29}{165150720}+\frac{1040537}{1902536294400}\cos(t)-\frac{1081}{3715891200}\cos(2t)\right.\\
&+\frac{3559}{634178764800}\cos(3t)+\frac{281}{928972800}\cos(4t)-\frac{4463}{317089382400}\cos(5t)\\
&-\frac{1}{58060800}\cos(6t)+\frac{1}{530841600}\cos(7t)-\frac{9941}{29727129600}\sin(t)\\
&-\frac{1402573}{1268357529600}\sin(2t)+\frac{10583}{29727129600}\sin(3t)-\frac{12499}{951268147200}\sin(4t)\\
&\left.-\frac{1}{11612160}\sin(5t)+\frac{73}{9909043200}\sin(6t)-\frac{1}{3715891200}\sin(8t)\right).
\end{aligned}
$$

The MADM solution closely approximates

$$u(x,t)=\cos\left(t-\frac{x}{16}\right)\left(1-\sin\left(t-\frac{x}{16}\right)\right),$$

which is given in [24] as an exact solution.

x	u_MADM(x, t=10)	u_exact(x, t=10)	Erorr
0.	-1.29554	-1.29554	0.
0.01	-1.29563	-1.29563	$6.536282626257162\times10^{-11}$
0.02	-1.29571	-1.29571	$1.0433933717024502\times10^{-9}$
0.03	-1.29579	-1.29579	$5.269929381768179\times10^{-9}$
0.04	-1.29588	-1.29588	$1.6616754638931752\times10^{-8}$
0.05	-1.29596	-1.29596	$4.047320856237491\times10^{-8}$
0.06	-1.29604	-1.29604	$8.372764659370091\times10^{-8}$
0.07	-1.29611	-1.29611	$1.547487631370359\times10^{-7}$
0.08	-1.29619	-1.29619	$2.633667790519212\times10^{-7}$
0.09	-1.29627	-1.29627	$4.2085449036832756\times10^{-7}$
0.1	-1.29634	-1.29634	$6.399081864394418\times10^{-7}$

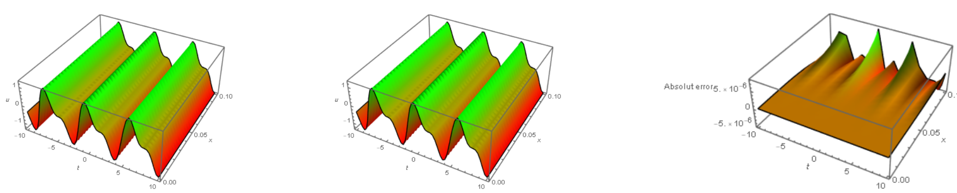
Table 1. Comparison between the exact solution and the MADM approximation at $t = 10$ and small x.

t	Exact	MADM	ADM [24]
0.	1.00000000000000	1.0000000000000	1.0000000000000
0.0002	00.999799980005333	0.999799980005333	0.999988278219568
0.0004	0.999599920042668	0.999599920042668	0.999976556481800
0.0006	0.999399820144005	0.999399820144005	0.999964834786718
0.0008	0.999199680341350	0.999199680341351	0.999953113134345
0.001	0.998999500666708	0.998999500666708	0.999941391524700

Table 2. Comparison between the exact solution, MADM approximation and the ADM approximation at $x = 0$ and small t.

t	Absolute Error (MADM)	Absolute Error (ADM) [24]
0	0.000000000000000	0.00000000000
0.0002	0.000000000000000	$1.882982142341616\times10^{-4}$
0.0004	$1.110223024625157\times10^{-16}$	$3.766364391323274\times10^{-4}$
0.0006	0.000000000000000	$5.650146427130798\times10^{-4}$
0.0008	$1.110223024625157\times10^{-16}$	$7.534327929941131\times10^{-4}$
0.001	$1.110223024625157\times10^{-16}$	$9.418908579912344\times10^{-4}$

Table 3. Absolute error between the exact solution, ADM, MADM based on the results shown in Table 2.

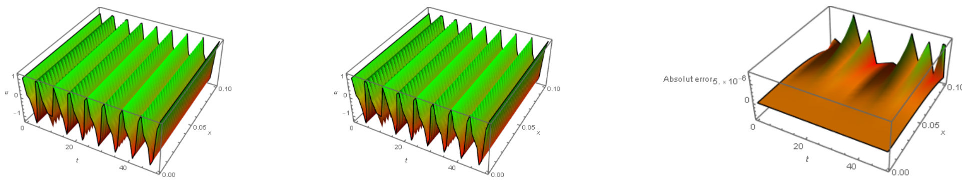


(a) **(b)** **(c)**

Figure 1. Surface plots of the solution for Example 5.1 over the interval $t \in [-10, 10]$: (a) Exact solution, (b) Fourth-order approximate solution using MADM, (c) Absolute error between the exact and MADM solutions.

Example 5.2. We solve Eq.(2.1) when $\alpha = 4$ and $\beta = \gamma = 1$ [25], by applying the steps in section 2. The components u_0, u_1, u_2 are

$$\begin{aligned}
u_0 =& 9 + c - 15kx \operatorname{sech}^4(k(ct + y_0))(\cosh(k(ct + y_0)) - 3\sinh(k(ct + y_0))) \\
&\times (\cosh(k(ct + y_0)) + \sinh(k(ct + y_0))) + \frac{15}{2}k^2x^2 \operatorname{sech}^5(k(ct + y_0)) \\
&\times (\cosh(k(ct + y_0)) + \sinh(k(ct + y_0)))(-7 + 5\cosh(2k(ct + y_0)) \\
&- 3\sinh(2k(ct + y_0))) - \frac{5}{8}k^2x^4 \operatorname{sech}^5(k(ct + y_0))(\cosh(k(ct + y_0)) \\
&+ \sinh(k(ct + y_0))) \times (-7 + 5\cosh(2k(ct + y_0)) - 3\sinh(2k(ct + y_0)))
\end{aligned}$$



(a) (b) (c)

Figure 2. The surface plot illustrates the solution for Example 5.1 over the interval $t \in [0, 50]$: (a) Exact solution, (b) Third-order approximation using the Modified Adomian Decomposition Method (MADM), and (c) Absolute error between the exact solution and the MADM approximation.

$$
\begin{aligned}
&+\frac{5}{2}k^3x^3\operatorname{sech}^6(k(ct+y_0))(\cosh(k(ct+y_0))+\sinh(k(ct+y_0)))\\
&\times(15\cosh(k(ct+y_0))-7\cosh(3k(ct+y_0))-51\sinh(k(ct+y_0))\\
&+9\sinh(3k(ct+y_0)))-\frac{1}{8}k^3x^5\operatorname{sech}^6(k(ct+y_0))(\cosh(k(ct+y_0))\\
&+\sinh(k(ct+y_0)))\times(15\cosh(k(ct+y_0))-7\cosh(3k(ct+y_0))\\
&-51\sinh(k(ct+y_0))+9\sinh(3k(ct+y_0)))\\
&-15\tanh(k(ct+y_0))(-1+\tanh(k(ct+y_0))+\tanh^2(k(ct+y_0))),
\end{aligned}
$$

$$
\begin{aligned}
u_1 =& -\frac{1}{64}kx^4\operatorname{sech}^8(k(ct+y_0))(\cosh(k(ct+y_0))+\sinh(k(ct+y_0)))\\
&\times\Big[(-750+380k^2-206kx)\cosh(k(ct+y_0))+(375+10k^2+233kx)\cosh(3k(ct+y_0))\\
&+15\cosh(5k(ct+y_0))-70k^2\cosh(5k(ct+y_0))-35kx\cosh(5k(ct+y_0))\\
&+210\sinh(k(ct+y_0))-420k^2\sinh(k(ct+y_0))-2238kx\sinh(k(ct+y_0))\\
&+165\sinh(3k(ct+y_0))-330k^2\sinh(3k(ct+y_0))+303kx\sinh(3k(ct+y_0))\\
&-45\sinh(5k(ct+y_0))+90k^2\sinh(5k(ct+y_0))+21kx\sinh(5k(ct+y_0))\Big],
\end{aligned}
$$

$$
\begin{aligned}
u_2 =& \frac{1}{32}kx^5\operatorname{sech}^6(k(ct+y_0))\Bigg[\left(\frac{8}{9}+2k^2\right)\cosh(2k(ct+y_0))\\
&+\left(3-14k^2\right)\cosh(4k(ct+y_0))\\
&+3(-37+10k^2+14-28k^2)\sinh(2k(ct+y_0))+(-3+6k^2)\sinh(4k(ct+y_0))\Bigg]\\
&(1+\tanh(k(ct+y_0))).
\end{aligned}
$$

$$
\begin{aligned}
u(x,t) =& 9+c-15\tanh(k(ct+y_0))-1+\tanh(k(ct+y_0))+\tanh^2(k(ct+y_0))\\
&-15kx\operatorname{sech}^4(k(ct+y_0))\left(\cosh(k(ct+y_0))-3\sinh(k(ct+y_0))\right)\\
&\times\left(\cosh(k(ct+y_0))+\sinh(k(ct+y_0))\right)\\
&+\frac{15}{2}k^2x^2\operatorname{sech}^5(k(ct+y_0))\left(\cosh(k(ct+y_0))+\sinh(k(ct+y_0))\right)\\
&\times\left(-7+5\cosh(2k(ct+y_0))-3\sinh(2k(ct+y_0))\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{5}{2}k^3x^3\operatorname{sech}^6(k(ct+y_0))\left(\cosh(k(ct+y_0))+\sinh(k(ct+y_0))\right)\\
&\times\left(15\cosh(k(ct+y_0))-7\cosh(3k(ct+y_0))-51\sinh(k(ct+y_0))\right.\\
&\left.+9\sinh(3k(ct+y_0))\right)-\frac{5}{64}x^4\operatorname{sech}^4(k(ct+y_0))(1+\tanh(k(ct+y_0)))\\
&\times\big(-56k^2+40k^2\cosh(2k(ct+y_0))-150k\operatorname{sech}^2(k(ct+y_0))\\
&+76k^3\operatorname{sech}^2(k(ct+y_0))+75k\cosh(3k(ct+y_0))\operatorname{sech}^3(k(ct+y_0))\\
&+2k^3\cosh(3k(ct+y_0))\operatorname{sech}^3(k(ct+y_0))+3k\cosh(5k(ct+y_0))\operatorname{sech}^3(k(ct+y_0))\\
&-14k^3\cosh(5k(ct+y_0))\operatorname{sech}^3(k(ct+y_0))-24k^2\sinh(2k(ct+y_0))\\
&+33k\operatorname{sech}^3(k(ct+y_0))\sinh(3k(ct+y_0))-66k^3\operatorname{sech}^3(k(ct+y_0))\sinh(3k(ct+y_0))\\
&-9k\operatorname{sech}^3(k(ct+y_0))\sinh(5k(ct+y_0))+18k^3\operatorname{sech}^3(k(ct+y_0))\sinh(5k(ct+y_0))\\
&+42k\operatorname{sech}^2(k(ct+y_0))\tanh(k(ct+y_0))-84k^3\operatorname{sech}^2(k(ct+y_0))\tanh(k(ct+y_0))\big)\\
&+\cdots.
\end{aligned}
$$

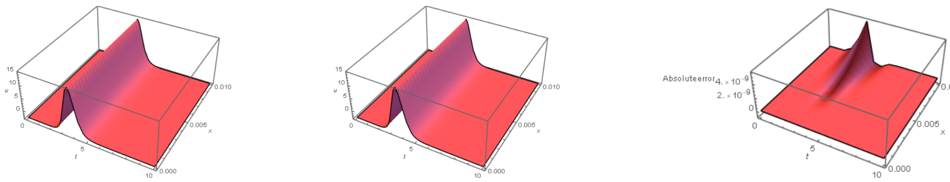
The MADM solution closely approximates

$$
\begin{aligned}
u(x,t)=&9+c-15\big(\tanh\left(k\left(-\mathrm{ct}+x-y_0\right)\right)+\tanh^2\left(k\left(-\mathrm{ct}+x-y_0\right)\right)\\
&-\tanh^3\left(k\left(-\mathrm{ct}+x-y_0\right)\right)\big),
\end{aligned}
$$

which is given in [25] as an exact solution.

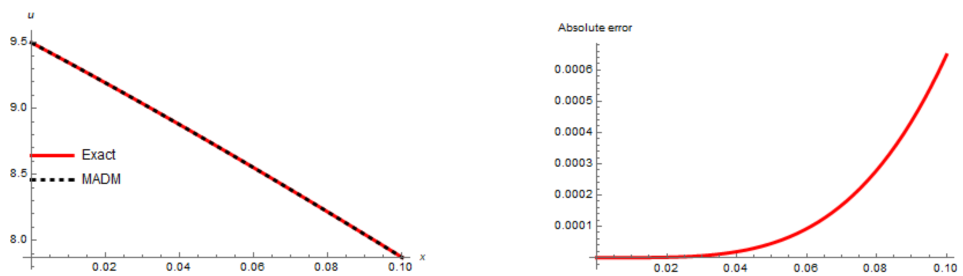
x	u_MADM(x, t=10)	u_exact(x, t=10)	Absolute Erorr
0.000000000	-2.999999751	-2.999999751	-3.552713679 $\times 10^{-15}$
0.100000000	-2.999999796	-2.999999796	2.348699013 $\times 10^{-11}$
0.200000000	-2.999999834	-2.999999833	3.541877902 $\times 10^{-10}$
0.300000000	-2.999999865	-2.999999863	1.686982998 $\times 10^{-9}$
0.400000000	-2.999999893	-2.999999888	5.005212245 $\times 10^{-9}$
0.500000000	-2.999999920	-2.999999908	1.144248252 $\times 10^{-8}$
0.600000000	-2.999999947	-2.999999925	2.215327299 $\times 10^{-8}$
0.700000000	-2.999999977	-2.999999939	3.819076078 $\times 10^{-8}$
0.800000000	-3.000000010	-2.999999950	6.039058142 $\times 10^{-8}$
0.900000000	-3.000000048	-2.999999959	8.925939099 $\times 10^{-8}$
1.000000000	-3.000000091	-2.999999966	1.248674302 $\times 10^{-7}$

Table 4. Comparison between the exact solution and the MADM approximation at $t = 10$, small x and $c = 0.5, k = 1, y_0 = -10$.



(a) (b) (c)

Figure 3. Surface plots of the solution for Example 5.2 with parameters $k = \frac{1}{2}, c = 3, y_0 = -10$, over the interval $t \in [0, 10]$: (a) Exact solution, (b) Second-order approximation using MADM, (c) Absolute error between the exact solution and the MADM approximation.



(a) (b)

Figure 4. The figures show that the solution $u(x, t)$ for Example 5.2 with parameters $c = 0.5, k = 1, y_0 = -10$ and $t = 20$: (a) Exact and MADM solutions (b) Absolute error between Exact Solution and MADM solution

6. Results and discussion

The numerical results presented in Figures 1 to 4 illustrate the performance and accuracy of the Modified Adomian Decomposition Method (MADM) for solving nonlinear problems in Examples 5.1 and 5.2.

In Figure 1, we analyze the behavior of the solution to Example 5.1 for *(*$\beta = \alpha = 1,$ *and* $\gamma = 0$*)*. Subplot (a) shows the exact solution, subplot (b) presents the MADM approximation, and subplot (c) illustrates the absolute error between them. It is evident that the MADM solution closely follows the exact profile, with minimal error. Furthermore, increasing the number of iterations can further extend the solution domain to *(*$t \in [-10, 10]$ *)*, while preserving convergence and accuracy.

Figure 2 shows the solution for the same example, but for a wider time domain *(*$t \in [0, 50]$*)*. The subfigures again show (a) the exact solution, (b) the MADM solution, and (c) the absolute error. As expected, the approximation remains consistent with the exact solution, confirming the stability of MADM over extended temporal intervals.

In Figure 3, we consider Example 5.2 with parameters *(*$\alpha = 4$ *and* $\beta = \gamma = 1$*)*, and initial condition *(*$k = \frac{1}{2}, c = 3, y_0 = -10$ *and* $t \in [0, 10]$*)*. The solution demonstrates MADM's robustness in handling varied parameter values, maintaining good agreement with the exact solution.

Figure 4 further examines Example 5.2 with *(*$c = 0.5, k = 1, y_0 = -10$ *and* $t = 20$*)*. Subplot (a) overlays the exact and MADM solutions, showing almost indistinguishable curves, while subplot (b) presents the absolute error, which remains minimal throughout the domain.

These results collectively validate the effectiveness of MADM in solving nonlinear problems with strong accuracy and convergence, even over large spatial and temporal domains.

7. Conclusion

Through the application of MADM, we successfully obtained an analytical solution for the KSE in the domain $t \in \mathbb{R}$ and $0 \leq x < 1$. The conducted numerical experiments and displayed convergence figures indicate that the MADM not only enhances computational efficiency but also maintains a high level of precision in approximation. Our findings illustrate that the method's iterative process yields rapid convergence, effectively handling the nonlinear aspects of the equation.

Moreover, the study reaffirms the reliability of the Modified Adomian Decomposition Method as a robust analytical tool for solving complex nonlinear equations in mathematical physics. Future research may focus on applying the method to other similar equations, potentially expanding its applicability and improving its computational techniques further. Overall, the results substantiate the potential of the Modified Adomian Decomposition Method as a powerful approach for deriving analytical solutions, paving the way for deeper exploration of chaotic and ordered dynamics in nonlinear systems.

Given its promising performance, future research could explore extending MADM to more challenging classes of problems, such as multi-dimensional nonlinear differential and integro-differential equations. Such extensions may provide deeper insights into high-dimensional chaotic systems and broaden the applicability of decomposition-based analytical approaches across various scientific and engineering domains.

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