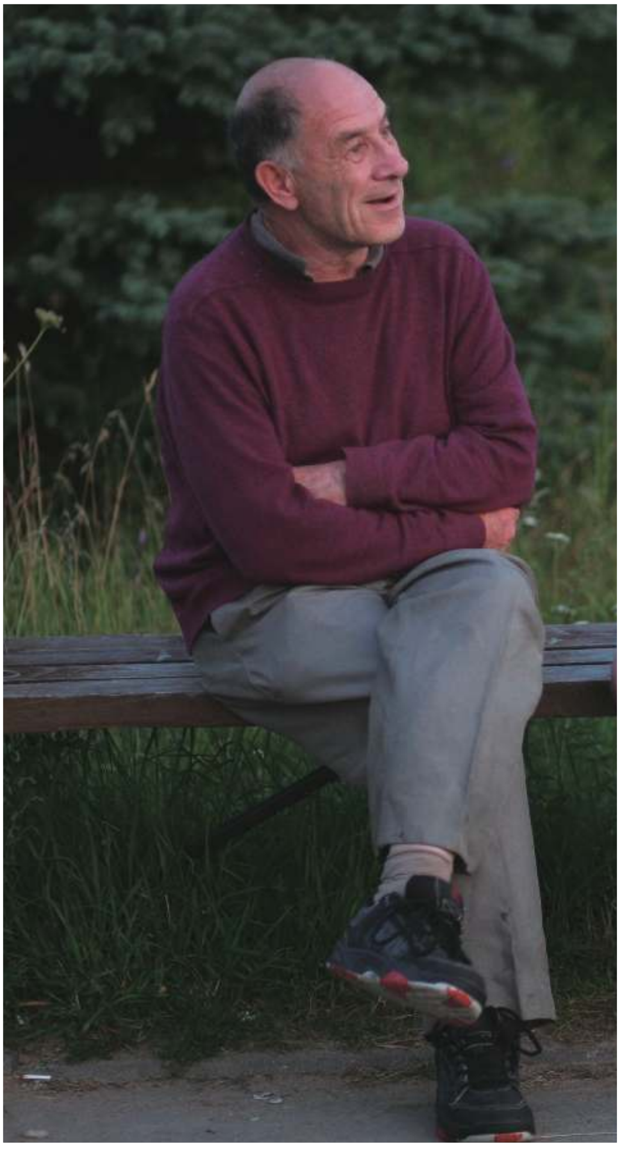
论数学教育[1]



Vladimir Arnold / 文
欧阳顺湘 / 译注

[1] 作者为著名的俄罗斯数学家弗拉基米尔•阿诺德（Vladimir Igorevich Arnold, 1937-2010）。他是 20 世纪最伟大的数学家之一，曾获克拉福德奖（1982）、沃尔夫奖（2001）和邵逸夫奖（2008）等奖。此文为 1997 年 3 月 7 日作者在法国巴黎探索文化宫（Palais de la Découverte，为法国科学教育中心博物馆）所发表演讲的扩充版，以俄文发表于 Uspekhi Mat. Nauk 53 (1998), no. 1 (319), 229-234; 英译见 Russian Math. Surveys 53 (1998), no. 1,229-236。英译网页版本可参 http://pauli.uni-muenster.de/~munsteg/arnold.html。亦有中译流传于网络。然而此中译不仅有笔误、漏译等不完善处，错也不少。因该中译译者佚名，现遵《数学文化》主编之托，参考该中译，根据英译重译该文。英译中也有不清晰处，是故徐佩教授也帮助参考了俄文。译文中尽可能为初学者可能不熟悉的部分、较易说清楚的内容加了些注释并配图以方便阅读。翻译本非易事，本文涉及的数学内容又很广，错误、不恰当处请读者批评。最后，关于阿诺德的故事以及与本文类似的观点的更多阐述，有兴趣的读者可阅读他于 1999 年春意外受伤后养病期间撰写的书 *Yesterday and Long Ago*（原书为俄文，2006 年出版；英译 2007 年由 Springer 出版）。

这篇文章反映了阿诺德对布尔巴基的批判，对庞加莱（Jules Henri Poincaré, 1854-1912）直觉主义的支持。值得指出的是，今年恰为庞加莱逝世 100 周年。

图 1 获 2008 年邵逸夫数学科学奖时的阿诺德（图片来源：http://shawprize.org）

数学是物理学的一部分。物理学是一门实验科学，是自然科学的一部分。而数学乃是物理学中实验代价较小的部分。

雅可比恒等式（蕴涵垂心定理：三角形的三条高相交于一点）[2] 如同地球是圆的（即同胚于球体）一样，是一个实验事实，只不过发现前者不那么昂贵。

20 世纪中叶，人们试图割裂物理学与数学。其后果已被证明是灾难性的。整整几代数学家在对其所从事科学之另

[2] 阿诺德后来有文章更具体地谈到这个观点。例如，在他写的 *A. N. Kolmogorov and natural science* (Russian Mathematical Surveys, 2004, 59:1, 27–46）一文中，即明确提到由雅可比恒等式 $[(a, b)], c]+[(b, c), a]+[(c, a), b] =0$ 可得出三角形三条高相交于垂心，其中 $[\cdot\ ,\cdot]$ 为向量积。(因此中学课程就应该学习该恒等式，不必要等到学李代数等较高等课程时才学习。）更多介绍可以参考《美国数学月刊》上 Nikolai V. Ivanov 的文章 *Arnol′d, the Jacobi Identity, and Orthocenters,* The American Mathematical Monthly, 2011, 118:1, 41–65。顺便提一件轶事。爱因斯坦的传记中记述过他小时两件“奇迹”。一是 5，6 岁时对指南针感兴趣，二是 12 岁时对欧几里得几何着迷，特别提到垂心定理。

Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap.

The Jacobi identity (which forces the heights of a triangle to cross at one point) is an experimental fact in the same way as that the Earth is round (that is, homeomorphic to a ball). But it can be discovered with less expense.

In the middle of the twentieth century it was attempted to divide physics and mathematics. The consequences turned out to be catastrophic. Whole generations of mathematicians grew up without knowing half of their science and, of course, in total ignorance of any other sciences. They first began teaching their ugly scholastic pseudo-mathematics to their students, then to schoolchildren (forgetting Hardy's warning that ugly mathematics has no permanent place under the Sun).

Since scholastic mathematics that is cut off from physics is fit neither for teaching nor for application in any other science, the result was the universal hate towards mathematicians - both

图 2 法国巴黎探索皇宫

图 3 法国巴黎高等师范学院

一半极其无知的情况下成长，遑论其他科学了。这些人先是把他们丑陋的学院式伪数学传给他们的弟子，接着这些丑陋的伪数学又被教给中小学校里的孩子们（他们浑然忘却了哈代的警告：丑陋的数学在世上无永存之地[3]）。

学院式数学脱离物理，既于教学无益，又对其他科学无用武之地，其后果是人们对数学家的普遍怨恨。 这样的人有学校里那些可怜的孩子们（他们当中有的可能还会成为将来的部长），也有应用这些数学的人。

由那些既无法掌握物理学又困倦于自卑中的半桶水式数学家们所拉起来的丑陋建筑，总使人想起“奇数的严格公理化理论”。显然，完全可以创造一种能够使得学生们称赞其完美无暇、内部结构和谐统一的理论（例如，可定义奇数个项的和以及任意多个因子的乘积）。按此狭隘观点，偶数要么被认为是“异端”，要么以后被当作“理想”对象补充入该理论，以此来应付物理与现实世界的需要。

不幸的是，数十年来，正是如上述这样丑陋扭曲的数学结构充斥着我们的数学教育。它肇始自法国，很快传染到基础数学教学，先是毒害大学生，接着中小学生也难免此灾（而灾区最先是法国，接着是其他国家，包括俄罗斯）。

倘若你问法国小学生，“2＋3 等于几”，他会这样回答：“3+2，因为加法适合交换律”。他不知道其和为几，甚至不

[3] 哈代（G. H. Hardy, 1877-1947）为英国著名数学家。该引语出自他的名著《一个数学家的自白》（*A Mathematician's Apology*, Cambridge University Press, 1994）。整段表达为：正像画家和诗人的模式一样，数学家的模式也必须是优美的；正像色彩和文字一样，数学家的思想也必须和谐一致。优美是第一关：丑陋的数学在世上无永存之地。（The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.）

on the part of the poor schoolchildren (some of whom in the meantime became ministers) and of the users.

The ugly building, built by undereducated mathematicians who were exhausted by their inferiority complex and who were unable to make themselves familiar with physics, reminds one of the rigorous axiomatic theory of odd numbers. Obviously, it is possible to create such a theory and make pupils admire the perfection and internal consistency of the resulting structure (in which, for example, the sum of an odd number of terms and the product of any number of factors are defined). From this sectarian point of view, even numbers could either be declared a heresy or, with passage of time, be introduced into the theory supplemented with a few “ideal” objects (in order to comply with the needs of physics and the real world).

Unfortunately, it was an ugly twisted construction of mathematics like the one above which predominated in the teaching of mathematics for decades. Having originated in France, this pervertedness quickly spread to teaching of foundations of mathematics, first to university students, then to school pupils of all lines (first in France, then in other countries, including Russia).

To the question “what is 2 + 3” a French primary school pupil replied: “3 + 2, since addition is commutative” . He did not know what the sum was equal to and could not even understand what he was asked about!

Another French pupil (quite rational, in my opinion) defined mathematics as follows: “there is a square, but that still has to be proved” .

Judging by my teaching experience in France, the university students' idea of mathematics (even of those taught mathematics

能理解你的问题是什么！

还有的法国小学生会如下阐述数学（我认为很有可能）：“存在一个正方形，但仍需证明。”

根据我本人在法国的教学经验，大学生们对数学的认知与这些小学生们同样槽糕（甚至包括那些在“高等师范学校”［高师］[4] 里学习数学的学生——我为这些明显很聪明但却被毒害颇深的孩子们感到极度的惋惜）。

譬如，这些学生从未见过一个抛物面。诸如描述由方程 $xy = z^2$ 所确定曲面之形状这样的问题即可使高师的数学家们发怵。作出平面上由参数方程（如 $x = t^3 - 3t, y = t^4 - 2t^2$）刻画的曲线是学生（甚至或许对大多数法国数学教授）完全无法做的问题。

从洛必达的第一部微积分教科书（名字即为“用于理解曲线的微积分”）[5] 开始，大致到古尔萨写的课本 [6]，解决这些问题的能力（和熟悉单位数乘法表一样）一直都被认为是每一个数学家应当具备的基本技能。

弱智的“抽象数学”的狂热者将几何（由此物理和现实的联系能常在数学中反映）统统摒除于教学之外。由古尔萨、埃尔米特、皮卡等人写的微积分教程被认为是过时而有害的，最近差点被巴黎第 6 和第 7 大学的学生图书馆当垃圾丢掉，只是在我的干预下才得以保存。

高师的学生，听完所有微分几何与代数几何课程（由有名望的数学家所教），却既不熟悉由椭圆曲线 $y^2 = x^3 + ax + b$ 决定的黎曼曲面，事实上也不知道曲面的拓扑分类（更不用提第一类椭圆积分以及椭圆曲线的群性质，即欧拉 - 阿贝尔加法定理了）。他们仅仅学到了霍奇结构与雅可比簇！

这样的事情怎么会在法国发生呢？！这可是为我们这个世界贡献了拉格朗日与拉普拉斯、柯西与庞加莱、勒雷与托姆这样的大家的国度啊！我觉得一个合理的解释出自彼德罗夫斯基 [7]。他在 1966 年曾教导过我：真正的数学家决不会拉帮结派，唯有弱者才会结党营生。他们可能因各种原因而联

[4] 高师是法国最好的大学，法国最具选拔性和挑战性的高等教育研究机构。校友中迄今已有施瓦茨、托姆以及新近的吴宝珠、维拉尼等共 10 位菲尔兹奖得主。

[5] 洛必达（Guillaume François Antoine, Marquis de l'Hôpital, 1661-1704），法国数学家。所提他撰写的教材原文名为 *Analyse des infiniment petits pour l´intelligence des lignes courbes*，1696 年出版。著名的洛必达法则即出现在此书中。

[6] 古尔萨（Edouard Goursat, 1858-1936），法国数学家。他的课本指 1902 年开始陆续出版的三卷本《数学分析教程》（*Cours d´analyse mathématique*）。中译最早有 19 世纪 30 年代王尚济的《解析数学讲义》以及 40 年代刘景芳翻译的《数学分析教程》。

[7] 彼德罗夫斯基（Ivan Georgievich Petrovsky, 1901-1973）研究偏微分方程等，对希尔伯特第 19 和第 16 问题有重要贡献。曾任莫斯科国立大学校长（期间阿诺德被录取）。

图 4 古尔萨

at the École Normale Supérieure - I feel sorry most of all for these obviously intelligent but deformed kids) is as poor as that of this pupil.

For example, these students have never seen a paraboloid and a question on the form of the surface given by the equation $xy = z^2$ puts the mathematicians studying at ENS into a stupor. Drawing a curve given by parametric equations (like $x = t^3 - 3t, y = t^4 - 2t^2$) on a plane is a totally impossible problem for students (and, probably, even for most French professors of mathematics).

Beginning with l'Hôpital's first textbook on calculus (“calculus for understanding of curved lines”) and roughly until Goursat´s textbook, the ability to solve such problems was considered to be (along with the knowledge of the times table) a necessary part of the craft of every mathematician.

Mentally challenged zealots of “abstract mathematics” threw all the geometry (through which connection with physics and reality most often takes place in mathematics) out of teaching. Calculus textbooks by Goursat, Hermite, Picard were recently dumped by the student library of the Universities Paris 6 and 7 (Jussieu) as obsolete and, therefore, harmful (they were only rescued by my intervention).

ENS students who have sat through courses on differential and algebraic geometry (read by respected mathematicians) turned out be acquainted neither with the Riemann surface of an elliptic curve $y^2 = x^3 + ax + b$ nor, in fact, with the topological classification of surfaces (not even mentioning elliptic integrals

图 5 1973 年发行的纪念彼特洛夫斯基的邮票

合（可能为超抽象，反犹主义或为“应用的和工业上的”问题），但其本质总是在一些非数学的社会问题中求生存。

我在此顺便提醒大家温习路易•巴斯德[8]的忠告：从来没有所谓的“应用科学”，有的只是科学的应用（而且非常实际的应用！）。

当时我一直对彼德罗夫斯基的话心存疑虑，但现今我愈来愈坚信：他说的一点没错。可观的超抽象活动最终归结为以工业化的模式无耻地掠夺原创者的成果，然后系统地将这些成果归功于拙劣的推广者。就如美洲没有以哥伦布的名字命名一样，数学结果也几乎从未以它们真正的发现者来命名。

为避免被误引，我须声明，由于某些未知的缘故，我自己的成果从未被上述方式侵占，虽然这样的事情经常发生在我的老师（柯尔莫哥洛夫、彼德罗夫斯基、庞特里亚金、洛赫林）和我的学生身上。M. Berry 教授曾提出如下两个原理：

Arnold 原理：如果某概念出现了某人名，则该人必非发现此概念者。

Berry 原理：阿诺德原理适用于自身。

我们还是回到法国的数学教育上来。

在我为莫斯科国立大学数学与力学系一年级学生时，微积分课教师是集合论拓扑学家 L.A. 图马金。他认真地讲解古尔萨版的古典法式微积分教程。他告诉我们若相应黎曼面是球面，则有理函数沿着代数曲线的积分可以求出来；若相应黎曼

[8] 路易•巴斯德（Louis Pasteur, 1822-1895），法国微生物学家、化学家，微生物学的奠基人之一。巴斯德原文常见英译为“There are no such things as applied sciences, only applications of science”。

of first kind and the group property of an elliptic curve, that is, the Euler-Abel addition theorem). They were only taught Hodge structures and Jacobi varieties!

How could this happen in France, which gave the world Lagrange and Laplace, Cauchy and Poincaré, Leray and Thom? It seems to me that a reasonable explanation was given by I.G. Petrovskii, who taught me in 1966: genuine mathematicians do not gang up, but the weak need gangs in order to survive. They can unite on various grounds (it could be super-abstractness, anti-Semitism or “applied and industrial” problems), but the essence is always a solution of the social problem - survival in conditions of more literate surroundings.

By the way, I shall remind you of a warning of L. Pasteur: there never have been and never will be any “applied sciences”, there are only applications of sciences (quite useful ones!).

In those times I was treating Petrovskii's words with some doubt, but now I am being more and more convinced of how right he was. A considerable part of the super-abstract activity comes down simply to industrialising shameless grabbing of discoveries from discoverers and then systematically assigning them to epigons-generalizers. Similarly to the fact that America does not carry Columbus's name, mathematical results are almost never called by the names of their discoverers.

In order to avoid being misquoted, I have to note that my own achievements were for some unknown reason never expropriated in this way, although it always happened to both my teachers (Kolmogorov, Petrovskii, Pontryagin, Rokhlin) and my pupils. Prof. M. Berry once formulated the following two principles:

The Arnold Principle. If a notion bears a personal name, then this name is not the name of the discoverer.

The Berry Principle. The Arnold Principle is applicable to itself.

Let's return, however, to teaching of mathematics in France.

When I was a first-year student at the Faculty of Mechanics and Mathematics of the Moscow State University, the lectures on calculus were read by the set-theoretic topologist L.A. Tumarkin, who conscientiously retold the old classical calculus course of French type in the Goursat version. He told us that integrals of rational functions along an algebraic curve can be taken if the corresponding Riemann surface is a sphere and, generally speaking, cannot be taken if its genus is higher, and that for the sphericity it is enough to have a sufficiently large number of double points on the curve of a given degree (which forces the curve to be unicursal: it is possible to draw its real points on the