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Improved 3D Surface Reconstruction via the Method of Fundamental Solutions

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Abstract. A new mathematical model of the modified bi-Helmholtz equation is proposed for the reconstruction of 3D implicit surfaces using the method of fundamental solutions. In the algorithm, we also show how to properly determine the parameter so that the spurious surface can be avoided. The main attraction of the proposed method is its simplicity. Four examples for the surface reconstruction are presented to validate the proposed numerical model.

AMS subject classifications: 65N35, 65N99

Key words: Method of fundamental solutions, implicit surface, modified bi-Helmholtz equation, computer graphics, image reconstruction.

1. Introduction

Reconstructing a surface from a set of point-cloud data has been the subject of considerable research in computer graphics and Computer Aided Design. Over the years, there is a growing popularity of representing the 3D objects by implicit surface modeling [3, 4, 14, 16, 17] in which the reconstructed surface can be represented by a single function. As such, the tedious patching procedures can be avoided.

There is a growing interest on the implicit representation in the 3D surface reconstruction. Radial basis functions have been successfully applied to 3D surface reconstruction by fitting a function to the object surface [2–4]. In the numerical process, two additional data cloud points normal to the surface have to be created. Hence,

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instead of using the given N data points, the radial basis function approach requires 3N data points for fitting an implicit function to a surface. In addition, several parameters have to be carefully adjusted in order to perform the surface fitting properly. Another approach of using an implicit function for surface reconstruction is through the mathematical model of partial differential equations (PDEs).

Tankelevich *et al.* [16] proposed to reconstruct a 3D implicit surface by solving both harmonic and biharmonic equations. As such, no additional contours and no data on the normal are required. Unlike the traditional mesh-based methods such as the finite element method [1, 9], finite difference method [19], or finite volume method [12], the main numerical technique for solving these PDE models is the method of fundamental solutions (MFS) [6, 7, 10, 18] which is a boundary meshless method and has been widely employed to solve homogeneous differential equations due to the effectiveness and simplicity of the method. These PDE models work quite well for smooth surfaces. Unfortunately for more complicated geometries with a high degree of concavity or sharp corner edges, spurious surfaces become visible. The remedy is to insert additional augmented points near the surface outside the domain in order to remove these spurious surfaces. It is not trivial how to place these additional points for various objects with a high degree of concavity and sharp edges.

Inspired by the implicit surface model via PDE and the MFS as a numerical tool showed in [16], it is the purpose of this paper to propose a more advanced PDE model for the 3D surface reconstruction. In our proposed model, we consider a homogenous modified bi-Helmholtz equation, which is a fourth order PDE, as the mathematical model. As such, we do not need the augmented points and normals outside the surface domain as required by the radial basis functions [4]. Moreover, our proposed PDE model has the capability of removing the spurious surfaces for complicated configurations with sharp edges. The MFS will be the method of choice for solving the modified bi-Helmholtz equation. In this model, there is only one parameter to be determined and we have proposed precisely how to select this parameter for a good recovery of the original surfaces.

This paper is organized as follows. In Section 2, we propose an improved PDE model for the 3D surface reconstruction. In Section 3, we give a brief review of the MFS and use it as a numerical tool to solve the PDE model proposed in Section 2. In Section 4, five numerical examples with complicated configurations and high concavity are considered. After extensive numerical tests, we propose how to choose the parameter precisely for the successful surface reconstruction. Finally, in Section 5, some conclusions and ideas for further work are outlined.

2. Surface reconstruction via the MFS

The method of fundamental solutions (MFS) is a simple and effective numerical method for solving homogeneous partial differential equations when the fundamental solution of the governing equation is available. The MFS has been widely applied to solve various kinds of problems in science and engineering. Inspired by the work